EXTRACTION OF INTENSIVELY MONITORED EXPANSION JOINTS BY MULTI-STAGE MIXED MARKOV HAZARD MODEL

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ABSTRACT**:** It is necessary to identify infrastructures relatively deteriorating fast, and to monitor, repair and renew the infrastructures. However, regarding methods to extract the infrastructures from inspection data obtained through ordinary inspections, there is actually no systematized methodology. In this paper, the authors propose the multi-stage mixed Markov deterioration hazard model and its multi-hierarchical Bayesian estimation. Furthermore, the benchmarking analysis towards stratified deterioration speeds corresponding to decision making levels and the methodology to extract intensively monitored infrastructures on each level are proposed. In order to verify the effectiveness of the proposed methodology, empirical analysis is carried out using the visual inspection data of 10,689 expansion joints in 21 lines of actual highway. The authors first mentioned that the deterioration process significantly depends on (1) kind of expansion joint, (2) kind of surface layer pavement and (3) traffic volume, and clarified that the expected life span of the expansion joints is about 18 years and it varies about 5 years due to the above mentioned factors. Then, it was found that the expected life span of the expansion joints varies from about 9 years to about 55 years by considering the heterogeneity of each line. Furthermore, the authors clarified that the expect life span varies from about 5 years to more than 100 years in the fastest deteriorating line by considering the heterogeneity of each expansion joint. Finally, by using the estimated result, the authors carried out the relative evaluation of hazard rate and extract the intensively monitored expansion joints.

KEYWORDS: multi-hierarchical Bayesian estimation, mixed Markov hazard model, heterogeneity

1. INTRODUCTION

In the asset management of infrastructures, by conducting the decision making process for maintenance, which has been based on implicit knowledge, with explicit knowledge, it is expected to (1) fulfill accountability based on objective data and (2) pass technologies efficiently inside an organization (Kobayashi, 2010). In recent years, statistical deterioration prediction methods utilizing visual inspection data have remarkably been developed. Especially, the development of the Markov deterioration hazard model (Tsuda et al., 2006) accelerated the practical application of asset management. In addition, the mixed Markov deterioration hazard model (Obama et al., 2008 and Mizutani et al., 2013) was proposed by taking into account the heterogeneity of each structure or member, which exists in a deterioration process, and it became possible to benchmark the deterioration rate of infrastructure.

Meanwhile, if the sophistication of the coming asset management is aimed, it is necessary to consider the utilization of not only the visual inspection but also the monitoring using sensors. It is realistic that the monitoring is preferentially applied to the member deteriorating fast, and the characteristic of the monitoring differs from the characteristic of visual inspection obtaining the uniform information of all members. However, because the fundamental purpose of asset management is to fulfill the accountability of maintenance, it is necessary to specify the decision making process of installation of monitoring systems to practice the asset management. At this time, it is important to pay attention to the hierarchical relation in the decision making processes of asset management such as the installation of monitoring system, the extraction of intensively monitored members, the priority of repair and the budgetary allocation. The decision making processes of asset management follow the concrete order below. First, the long term maintenance plan is designed for the entire targeted infrastructures. Second, the consideration with macro perspective is carried out corresponding to deterioration characteristics of management office and region. Finally, through the consideration with micro perspective targeting infrastructure groups more fragmented in the management office and individual infrastructures, the decision making is conducted. Thus, in the actual asset management, it is required not to simultaneously evaluate the entire targeted infrastructure but to relatively evaluate the individual infrastructures with the multi hierarchical decision making processes.

Based on the above problem awareness, in this paper, the authors propose the multi-stage mixed Markov deterioration hazard model innovating hierarchical heterogeneity parameters and its estimation method. In addition, by relatively comparing deterioration rates of each member using the heterogeneity parameters, it becomes possible to extract intensively monitored members, for example, monitored by using sensors.

Chapter 2 outlines the multi-stage mixed Markov deterioration hazard model, and Chapter 3 describes a multi-hierarchical Bayesian estimation method in detail. Lastly, Chapter 4 empirically analyzes actual expansion joints by using visual inspection data.

2. MULTI-STAGE MIXED MARKOV HAZARD MODEL

2.1 Markov chain model

The deterioration process of infrastructure can be expressed with a Markov chain model using a transition probability matrix. The Markov deterioration hazard model (Tsuda et al., 2006) in order to estimate the Markov transition probability is proposed. Here, for reader's convenience, the outline of the Markov deterioration hazard model is described. The transition between the condition states of two time points can be expressed as the Markov transition probability. Consider the transition of states between two time points. The state at time τ_A is $h(\tau_A)$, and the state at time τ_B is $h(\tau_B)$. If $h(\tau_A)=i$ and $h(\tau_B)=j$, the Markov transition probability is $\text{Prob}[h(\tau_B)=j \mid h(\tau_A)=i]$. The condition for this Markov transition probability is that the state is i at time τ_A, and the conditional transition probability that the state will be j at time τ_B can be defined as:

$$\text{Prob}\left[h(\tau_B)=j \mid h(\tau_A)=i\right]=\pi_{ij} \quad (1)$$

By deriving the pair of states (i,j) from a transition probability in this way, we can also obtain a Markov transition probability matrix.

$$\Pi=\begin{pmatrix} \pi_{11} & \cdots & \pi_{1I} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \pi_{II} \end{pmatrix} \quad (2)$$

The Markov transition probability (1) expresses the transition probability between the two conditional time points τ_A and τ_B. Naturally, if the inspection intervals differ, the transition probability will also differ. As long as there are no repairs, deterioration proceeds constantly, so $\pi_{ij} = 0 \ (i > j)$ is true. Also, from the definition of the transition probability, $\sum_{j=i}^{I} \pi_{ij} = 1$ is true. In other words, regarding the Markov transition probability, the following must hold true.

$$\left.\begin{array}{l} \pi_{ij} \geq 0 \ (i, j = 1, \ldots, j) \\ \pi_{ij} = 0 \ (when \quad i > j) \\ \sum_{j=i}^{I} \pi_{ij} = 1 \end{array}\right\} \quad (3)$$

The condition state I is the absorbing state in the Markov chain as long as there are no repairs, and $\pi_{II} = 1$ is true. Moreover, the Markov transition probability is defined independently of past deterioration records. The Markov chain model satisfies the Markov property that, regardless of the time at which the state transitions from $i-1$ to i, the probability of the transition taking place between time τ_A and time τ_B depends only on the condition state at time τ_A.

2.2 Multi-stage mixed Markov hazard model

The purpose of this paper is the deterioration prediction of the individual members based on the visual inspection data. The entire members are divided into M stage infrastructure groups. The group is assessment unit. First, the entire members are divided into K_0 number of group k_1 (k_1=1,…,K_0) named the 1st stage group. Second, the members in the 1st stage group k_1 are divided into K_{k1} number of group k_2 (k_2=1,…, K_{k1}) named the 2nd stage group. Similarly, the members in the 2nd stage group k_2 are divided into $K_{k1,k2}$ number of group k_3 (k_3=1,…, $K_{k1,k2}$) named the 3rd stage group. Thus, the members in the m-1th stage group k_{m-1} (k_{m-1}=1,…,$K_{k1,\ldots,km-1}$) are divided into $K_{k1,\ldots,km-1}$ number of group k_m (k_m=1,…,$K_{k1,\ldots,km-1}$) named the mth stage group, and the division are repeated until m=M. At this time, the m(m=1,…,M)th stage group consists of $\sum_{k_1=1}^{K_0}\sum_{k_2=1}^{K_{k1}}\cdots\sum_{k_m=1}^{K_{k1,\ldots,km-1}} K_{k1,\ldots,km}$ number of groups, and the number of all groups K^{ε} are:

$$\begin{aligned} K^{\varepsilon} = K_0 + \sum_{k_1=1}^{K_0} (K_{k_1} + \sum_{k_2=1}^{K_{k_1}} (K_{k_1,k_2} \\ + \sum_{k_3=1}^{K_{k_1,k_2}} (K_{k_1,k_2,k_3} + \sum_{k_4=1}^{K_{k_1,k_2,k_3}} (K_{k_1,k_2,k_3,k_4} \\ \cdots \\ + \sum_{k_{M-1}=1}^{K_{k_1,\ldots,k_{M-2}}} (K_{k_1,\ldots,k_{M-1}} + \sum_{k_M=1}^{K_{k_1,\ldots,k_{M-1}}} K_{k_1,\ldots,k_M})\cdots)))) \end{aligned} \quad (4)$$

In addition, the Mth stage group k_M (k_M=1,…,$K_{k1,\ldots,kM-1}$) consists of $S_{k1,\ldots,kM}$ number of members.

Now, in order to establish a unique hazard rate of each mth stage group, heterogeneity parameter $\varepsilon_{km|k1,\ldots,km-1} (m = 1, \ldots, M; k_m = 1, \ldots, K_{k1,\ldots,km-1})$ are implemented. At this time, the state of member $s_{k1,\ldots,kM}$ of group k_1, k_2, …, k_M is i (i=1,…,I-1), and its hazard rate can be expressed as:

$$\begin{aligned} \lambda_i^{s_{k_1,\ldots,k_M}} = \tilde{\lambda}_i^{s_{k_1,\ldots,k_M}} \varepsilon_{k_1} \varepsilon_{k_2|k_1} \varepsilon_{k_3|k_1,k_2} \cdots \\ \varepsilon_{k_m|k_1,\ldots,k_{m-1}} \cdots \varepsilon_{k_M|k_1,\ldots,k_{M-1}} \end{aligned} \quad (5)$$

Here, $\tilde{\lambda}_i^{s_{k1,\ldots,kM}}$ is the standard hazard rate of state I for member $s_{k1,\ldots,kM}$. Heterogeneity parameter $\varepsilon_{km|k1,\ldots,km-1}$ is a random variable derived from gamma distribution $\bar{g}(\varepsilon_{km|k1,\ldots,km-1} \mid \phi_{k1,\ldots,km-1}^m)$ with average of 1 and variance of $1/\phi_{k1,\ldots,km-1}^m$.

$$\begin{aligned} &\bar{g}(\varepsilon_{k_m|k_1,\ldots,k_{m-1}} \mid \phi_{k_1,\ldots,k_{m-1}}^m) \\ &= \frac{\Phi_{k_1,\ldots,k_{m-1}}^m}{\Gamma(\phi_{k_1,\ldots,k_{m-1}}^m)} (\varepsilon_{k_m|k_1,\ldots,k_{m-1}})^{\phi_{k_1,\ldots,k_{m-1}}^m - 1} \\ &\cdot \exp(-\phi_{k_1,\ldots,k_{m-1}}^m \varepsilon_{k_m|k_1,\ldots,k_{m-1}}) \end{aligned} \quad (6)$$

Here, $\boldsymbol{\Phi}_{k1,\ldots,km-1}^m = (\phi_{k1,\ldots km-1}^m)^{\phi_{k1,\ldots km-1}^m}$, and $\phi_{k1,\ldots,km-1}^m = \phi_0^1$ when m=1. Variance parameter $\phi_{k1,\ldots,km-1}^m$ is defined in each

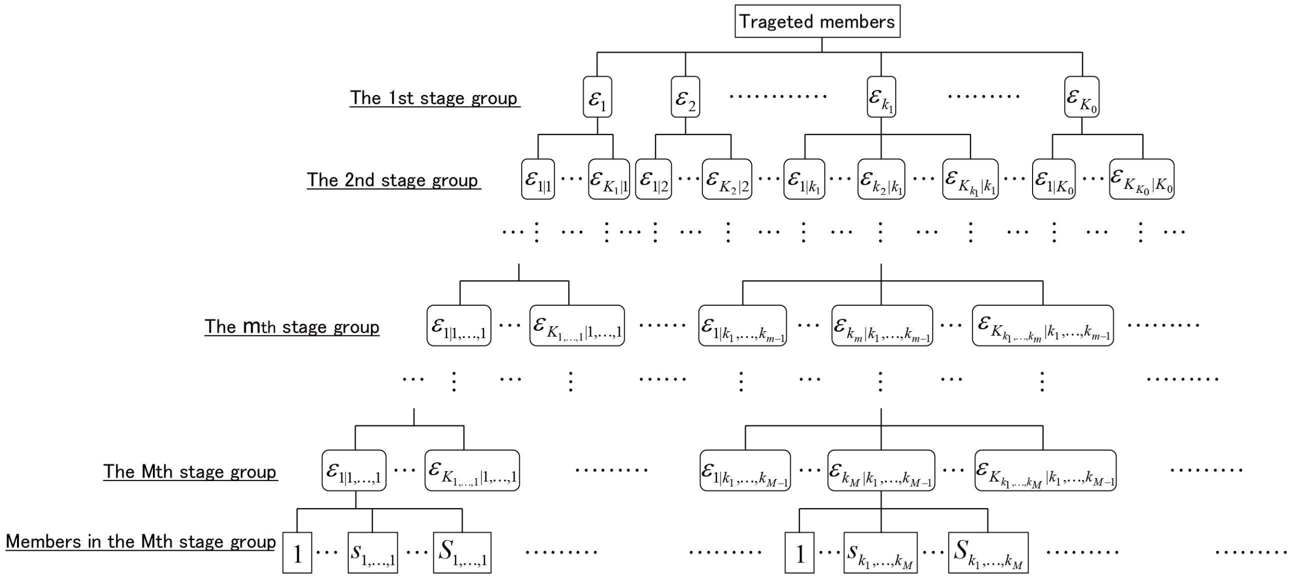


Figure 1 Hierarchy of heterogeneity

heterogeneity parameter group ($\varepsilon_{k1},\cdots,\varepsilon_{km|k1,\ldots,km-1}$), and the total number of $\phi^{m}_{k1,\ldots,km-1}$ is:

$$K^{\phi}=K_0+\sum_{k_1=1}^{K_0}(K_{k_1}+\sum_{k_2=1}^{K_{k_1}}K_{k_1,k_2}$$
$$+\sum_{k_3=1}^{K_{k_1,k_2}}(K_{k_1,k_2,k_3}+\sum_{k_4=1}^{K_{k_1,k_2,k_3}}(K_{k_1,k_2,k_3,k_4} \tag{7}$$
$$\cdots$$
$$+\sum_{k_{M-1}=1}^{K_{k_1,\ldots,k_{M-2}}}(K_{k_1,\ldots,k_{M-1}}+\sum_{k_M=1}^{K_{k_1,\ldots,k_{M-1}}}K_{k_1,\ldots,k_M})\cdots))))$$

The above mentioned hierarchical division of heterogeneity parameters is shown in Figure1.

Here, the heterogeneity parameter $\varepsilon_{km|k1,\ldots,km-1}$ of the mth stage group is fixed as $\bar{\varepsilon}_{km|k1,\ldots,km-1}$. In addition, the product of heterogeneity parameters is expressed as:

$$\bar{\varepsilon}_{k_1,\ldots,k_M}=\bar{\varepsilon}_{k_1}\bar{\varepsilon}_{k_2|k_1}\bar{\varepsilon}_{k_3|k_1,k_2}\cdots\bar{\varepsilon}_{k_m|k_1,\ldots,k_{m-1}}\cdots\bar{\varepsilon}_{k_M|k_1,\ldots,k_{M-1}} \tag{8}$$

At this time, the probability $\pi_{ii}(z^{s_{k1,\ldots,kM}}\mid\bar{\varepsilon}_{k1,\ldots,kM})$ that the state of an arbitrary member $s_{k1,\ldots,kM}$ will stay in condition state i at inspection $\tau_A^{s_{k1,\ldots,kM}}$, as well as the following inspection $\tau_B^{s_{k1,\ldots,kM}}=\tau_A^{s_{k1,\ldots,kM}}+z^{s_{k1,\ldots,kM}}$ can be expressed, using the hazard rate (equation 5), as (Lancaster, 1990):

$$\pi_{ii}(z^{s_{k_1,\ldots,k_M}}\mid\bar{\varepsilon}_{k_1,\ldots,k_m})=\exp(-\bar{\lambda}_i^{s_{k_1,\ldots,k_M}}z^{s_{k_1,\ldots,k_M}}) \tag{9}$$

However, $\bar{\lambda}_i^{s_{k1,\ldots,kM}}=\tilde{\lambda}_i^{s_{k1,\ldots,kM}}\bar{\varepsilon}_{k_1,\ldots,k_M}$ .Also, the probability $\pi_{ij}(z^{s_{k1,\ldots,kM}}\mid\bar{\varepsilon}_{k1,\ldots,kM})$ that the state will be i at inspection $\tau_A^{s_{k1,\ldots,kM}}$ and j at inspection $\tau_B^{s_{k1,\ldots,kM}}=\tau_A^{s_{k1,\ldots,kM}}+z^{s_{k1,\ldots,kM}}$ can be expressed as (Tsuda et al., 2006):

$$\pi_{ij}(z^{s_{k_1,\ldots,k_M}}\mid\bar{\varepsilon}_{k_1,\ldots,k_M})=\sum_{u=i}^{j}\prod_{t=i,\neq u}^{j-1}\frac{\tilde{\lambda}_t^{s_{k_1,\ldots,k_M}}}{\tilde{\lambda}_t^{s_{k_1,\ldots,k_M}}-\tilde{\lambda}_u^{s_{k_1,\ldots,k_M}}}\cdot\exp(-\bar{\lambda}_u^{s_{k_1,\ldots,k_M}}z^{s_{k_1,\ldots,k_M}})$$
$$=\sum_{u=i}^{j}\psi_{ij}^{u}(\tilde{\lambda}^{s_{k_1,\ldots,k_M}})\exp(-\bar{\lambda}_u^{s_{k_1,\ldots,k_M}}z^{s_{k_1,\ldots,k_M}}) \tag{10}$$

Due to the Markov transition probability's condition, the probability $\pi_{iI}(z^{s_{k1,\ldots,kM}}\mid\bar{\varepsilon}_{k1,\ldots,kM})$ can be expressed as:

$$\pi_{iI}(z^{s_{k_1,\ldots,k_M}}\mid\bar{\varepsilon}_{k_1,\ldots,k_M})=1-\sum_{j=i}^{I-1}\pi_{ij}(z^{s_{k_1,\ldots,k_M}}\mid\bar{\varepsilon}_{k_1,\ldots,k_M}) \tag{11}$$

However, $\tilde{\lambda}^{s_{k1,\ldots,kM}}=(\tilde{\lambda}_1^{s_{k1,\ldots,kM}},\cdots,\tilde{\lambda}_{I-1}^{s_{k1,\ldots,kM}})$ and, using the only standard hazard rate, $\psi_{ij}^{u}(\tilde{\lambda}^{s_{k1,\ldots,kM}})$ is defined as:

$$\psi_{ij}^{u}(\tilde{\lambda}^{s_{k_1,\ldots,k_M}})=\prod_{t=i,\neq u}^{j-1}\frac{\tilde{\lambda}_t^{s_{k_1,\ldots,k_M}}}{\tilde{\lambda}_t^{s_{k_1,\ldots,k_M}}-\tilde{\lambda}_u^{s_{k_1,\ldots,k_M}}} \tag{12}$$

2.3 Visual inspection data and hazard rate

Now, let us mention inspections are carried out at time $\tau_A^{s_{k1,\dots,kM}}$ and $\tau_B^{s_{k1,\dots,kM}} = \tau_A^{s_{k1,\dots,kM}} + z^{s_{k1,\dots,kM}}$ on the expansion joint $s_{k1,\dots,kM}$. The inspection sample of member $s_{k1,\dots,kM}$ includes the inspection interval $z^{s_{k1,\dots,kM}}$ and ratings $\bar{h}(\tau_A^{s_{k1,\dots,kM}})$ and $\bar{h}(\tau_B^{s_{k1,\dots,kM}})$. The symbol " ¯ " signifies an actual inspected value. Based on inspected ratings, the dummy variable $\bar{\delta}_{ij}^{s_{k1,\dots,kM}}$ is defined as:

$$\bar{\delta}_{ij}^{s_{k_1,\dots,kM}} = \begin{cases} 1 & when\ \bar{h}(\tau_A^{s_{k_1,\dots,kM}}) = i, \bar{h}(\tau_B^{s_{k_1,\dots,kM}}) = j \\ 0 & otherwise \end{cases} \quad (13)$$

Moreover, the dummy variable vector is expressed as $\bar{\boldsymbol{\delta}}^{s_{k1,\dots,kM}} = (\bar{\delta}_{11}^{s_{k1,\dots,kM}}, \cdots, \bar{\delta}_{I-1I}^{s_{k1,\dots,kM}})$, and the F dimensions characteristic variable vector is expressed as $\bar{\boldsymbol{x}}^{s_{k1,\dots,kM}} = (\bar{x}_1^{s_{k1,\dots,kM}}, \cdots, \bar{x}_F^{s_{k1,\dots,kM}})$. Also, the standard hazard rate $\tilde{\lambda}_i^{s_{k1,\dots,kM}}$ varies depending on characteristic variables, and the standard hazard rate $\tilde{\lambda}_i^{s_{k1,\dots,kM}}$ can be expressed, using characteristic variables, as:

$$\tilde{\lambda}_i^{s_{k_1,\dots,kM}} = \exp(\boldsymbol{x}^{s_{k_1,\dots,kM}} \boldsymbol{\beta}_i') \quad (14)$$

However, $\boldsymbol{\beta}_i = (\beta_{i,1}, \cdots, \beta_{i,F})$ is the row vector of unknown parameter, and the symbol " ' " signifies transposition. Because $x_1^{s_{k1,\dots,kM}} = 1$, $\beta_{i,1}$ is a constant term.

3. HIERARCHICAL BAYESIAN ESTIMATION

3.1 Hierarchical Bayesian estimation method

A infrastructure's visual inspection data are necessary in order to assess the heterogeneity of that specific infrastructure, but in general there are usually no adequate records of visual inspection data for individual infrastructures. Even under this situation, the mixed Markov deterioration hazard model suggested in this paper can be used to analyze the average deterioration process of all infrastructures as well as deterioration characteristics of the targeted infrastructure or infrastructure group from visual inspection data records for the same infrastructure group (assessment unit). In particular, the multi-stage mixed Markov hazard model assumes that the heterogeneity parameter of the mth stage group $\varepsilon_{k_m|k_1,\dots,k_M}$ is subject to a prior distribution expressed as a gamma distribution with a mean of 1 and variance of $1/\phi_{k1,\dots,kM}^m$. Furthermore, with hierarchical Bayesian estimation, we can establish a prior distribution for the heterogeneity parameter's variance parameter $\phi_{k1,\dots,kM}^m$ (hyper parameter). These models with hierarchical prior distributions are called hierarchical Bayesian models[4]. The method is studied mostly in marketing analysis. This paper also uses a hierarchical Bayesian model to estimate the mixed Markov deterioration hazard model.

Bayesian estimation[5] is an estimation method that uses a parameter's prior distribution and the likelihood function defined from observed data to estimate the parameter's posterior distribution. Now, the unknown parameter vector is $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\varphi}, \boldsymbol{\varepsilon})$ and the visual inspection data is $\boldsymbol{\Xi}$, therefore the likelihood function can be expressed as $L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$. If $\boldsymbol{\theta}$ is the random variable and it is subject to the prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$ when visual inspection data $\boldsymbol{\xi}$ is obtained, according to Bayes' theorem[6], can be expressed as:

$$\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi}) = \frac{L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})\boldsymbol{\pi}(\boldsymbol{\theta})}{\int_\Theta L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})\boldsymbol{\pi}(\boldsymbol{\theta})d\boldsymbol{\theta}} \quad (15)$$

However, Θ is the parameter space. At this time, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$ can be expressed as:

$$\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi}) \propto L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})\boldsymbol{\pi}(\boldsymbol{\theta}) \quad (16)$$

In general, Bayesian estimation is conducted in the order of: 1) Establish the parameter's prior probability density function $\pi(\theta)$ based on prior experience information, 2) Define the likelihood function $L(\Xi \mid \theta)$ using the obtained data, 3) Revise the prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ based on Bayes' theorem (14), and obtain the posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$ of parameter $\boldsymbol{\theta}$. The unknown parameter's prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ in the mixed Markov deterioration hazard model is:

$$\begin{aligned}\boldsymbol{\pi}(\boldsymbol{\theta}) &= \boldsymbol{\pi}(\boldsymbol{\beta},\boldsymbol{\varphi},\boldsymbol{\varepsilon}) = \boldsymbol{\pi}(\boldsymbol{\beta})\boldsymbol{\pi}(\boldsymbol{\varepsilon} \mid \boldsymbol{\varphi})\boldsymbol{\pi}(\boldsymbol{\varphi}) \\ &= \prod_{i=1}^{I-1}\boldsymbol{\pi}(\boldsymbol{\beta}_i)\boldsymbol{\pi}(\phi_0^1)\prod_{k_1=1}^{K_0}\{\boldsymbol{\pi}(\varepsilon_{k_1})\boldsymbol{\pi}(\phi_{k_1}^2) \\ &\cdot \prod_{k_2=1}^{K_{k_1}}\{\boldsymbol{\pi}(\varepsilon_{k_2|k_1})\boldsymbol{\pi}(\phi_{k_1,k_2}^3) \\ &\cdot \prod_{k_3=1}^{K_{k_1,k_2}}\{\boldsymbol{\pi}(\varepsilon_{k_3|k_1,k_2})\boldsymbol{\pi}(\phi_{k_1,k_2,k_3}^4)\cdots \\ &\cdot \prod_{k_M=1}^{K_{k_1,\dots,k_{M-1}}}\{\boldsymbol{\pi}(\varepsilon_{k_M|k_1,\dots,k_M})\}\cdots\}\}\}\end{aligned} \tag{17}$$

We can see that probability distribution of heterogeneity parameter ε in this paper's multi-stage mixed Markov deterioration hazard model and the prior distribution of parameter ϕ in the probability distribution have hierarchical structures. The multi hierarchical Bayesian estimation method establishes prior distributions for each unknown parameter $\boldsymbol{\theta} = (\boldsymbol{\beta},\boldsymbol{\varphi},\boldsymbol{\varepsilon})$, and calculates conditional posterior probability density functions for each parameter.

3.2 Formulation of posterior distributions

Let's say parameter $\boldsymbol{\theta} = (\boldsymbol{\beta},\boldsymbol{\phi},\boldsymbol{\varepsilon})$ is a given condition. At this time, the joint probability (likelihood) $L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$ of the visual inspection data $\boldsymbol{\Xi}$ is:

$$L(\boldsymbol{\theta} \mid \boldsymbol{\Xi}) = \prod_{i=1}^{I-1}\prod_{j=i}^{I}\prod_{k_1=1}^{K_0}\prod_{m=2}^{M}\prod_{k_m}^{K_{k_1,\dots,k_{m-1}}}\prod_{s_{k_1,\dots,k_M}=1}^{S_{k_1,\dots,k_M}} \{\pi_{ij}(\bar{z}^{s_{k_1,\dots,k_M}},\bar{\boldsymbol{x}}^{s_{k_1,\dots,k_M}} \mid \boldsymbol{\beta},\boldsymbol{\varphi},\boldsymbol{\varepsilon})\}^{\bar{\delta}_{ij}^{s_{k_1,\dots,k_M}}} \tag{18}$$

Also, the prior probability density functions of unknown parameters $\boldsymbol{\theta} = (\boldsymbol{\beta},\boldsymbol{\phi},\boldsymbol{\varepsilon})$ in equation (17) are as follows. First, a multi-dimensional normal distribution was used for prior probability density function $\boldsymbol{\pi}(\boldsymbol{\beta}_i)$ of unknown parameter $\boldsymbol{\beta}_i$. The probability density function of the F-dimensional normal distribution is derived from:

$$\begin{aligned}&\boldsymbol{\pi}(\boldsymbol{\beta}_i) \\ &= \frac{1}{(\sqrt{2\pi})^F}\cdot\exp\{-\frac{1}{2}(\boldsymbol{\beta}_i-\boldsymbol{\mu}_i)\boldsymbol{\Sigma}_i^{-1}(\boldsymbol{\beta}_i-\boldsymbol{\mu}_i)'\}\end{aligned} \tag{19}$$

However, $\boldsymbol{\mu}_i$ is an expected value vector and $\boldsymbol{\Sigma}_i$ is a variance-covariance matrix. The prior probability density function $\boldsymbol{\pi}(\boldsymbol{\varepsilon} \mid \boldsymbol{\varphi})$ of $\boldsymbol{\varepsilon}$ is already given in gamma distribution (equation 6). Furthermore, a gamma distribution $h(\phi_{k_1,\dots k_{m-1}}^m \mid \alpha_{k_1,\dots k_{m-1}}^m,\gamma_{k_1,\dots k_{m-1}}^m)$ is established for prior probability density function of equation (17)'s variance parameter $\boldsymbol{\varphi}$. Hence, the prior probability density function can be defined as:

$$\begin{aligned}&\boldsymbol{\pi}(\phi_{k_1,\dots,k_{m-1}}^m) \\ &= \frac{1}{(\gamma_{k_1,\dots,k_{m-1}}^m)^{\alpha_{k_1,\dots,k_{m-1}}^m}\Gamma(\alpha_{k_1,\dots,k_{m-1}}^m)} \\ &\cdot(\phi_{k_1,\dots,k_{m-1}}^m)^{\alpha_{k_1,\dots,k_{m-1}}^m-1}\exp(-\frac{\phi_{k_1,\dots,k_{m-1}}^m}{\gamma_{k_1,\dots,k_{m-1}}^m}) \\ &(m \geq 2)\end{aligned} \tag{20}$$

However, when m=1, the prior probability density function can be expressed as:

$$\begin{aligned}&\boldsymbol{\pi}(\phi_0^1) \\ &= \frac{1}{(\gamma_0^1)^{\alpha_0^1}\Gamma(\alpha_0^1)}(\phi_0^1)^{\alpha_0^1-1}\exp(-\frac{\phi_0^1}{\gamma_0^1})\end{aligned} \tag{21}$$

Therefore, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi})$ can be formulated as:

$$\begin{aligned}&\boldsymbol{\pi}(\boldsymbol{\theta} \mid \boldsymbol{\Xi}) \\ &\propto L(\boldsymbol{\theta} \mid \boldsymbol{\Xi})\boldsymbol{\pi}(\boldsymbol{\theta}) \\ &\propto \prod_{i=1}^{I-1}\prod_{j=i}^{I}\prod_{k_1=1}^{K_0}\prod_{m=2}^{M}\prod_{k_m}^{K_{k_1,\dots,k_{m-1}}}\prod_{s_{k_1,\dots,k_M}=1}^{S_{k_1,\dots,k_M}} \\ &\quad \{\pi_{ij}(\bar{z}^{s_{k_1,\dots,k_M}},\bar{\boldsymbol{x}}^{s_{k_1,\dots,k_M}} \mid \boldsymbol{\beta},\boldsymbol{\varphi},\boldsymbol{\varepsilon})\}^{\bar{\delta}_{ij}^{s_{k_1,\dots,k_M}}} \\ &\cdot \prod_{i=1}^{I-1}\boldsymbol{\pi}(\boldsymbol{\beta}_i)\boldsymbol{\pi}(\phi_0^1)\prod_{k_1=1}^{K_0}\{\boldsymbol{\pi}(\varepsilon_{k_1})\boldsymbol{\pi}(\phi_{k_1}^2) \\ &\cdot \prod_{k_2=1}^{K_{k_1}}\{\boldsymbol{\pi}(\varepsilon_{k_2|k_1})\boldsymbol{\pi}(\phi_{k_1,k_2}^3) \\ &\cdot \prod_{k_3=1}^{K_{k_1,k_2}}\{\boldsymbol{\pi}(\varepsilon_{k_3|k_1,k_2})\boldsymbol{\pi}(\phi_{k_1,k_2,k_3}^4)\cdots \\ &\cdot \prod_{k_M=1}^{K_{k_1,\dots,k_{M-1}}}\{\boldsymbol{\pi}(\varepsilon_{k_M|k_1,\dots,k_M})\}\cdots\}\}\}\end{aligned} \tag{22}$$

However, it is difficult to analytically estimate the joint probability density function (20) and acquire direct samples. Therefore, in this paper the unknown parameter vector $\boldsymbol{\theta}$ was estimated from a Markov chain Monte Carlo method (Gill, 2007 and Liang et al., 2010) (MCMC method). An MCMC method obtains samples from posterior distributions by repeated random generation of parameter $\boldsymbol{\theta}$ samples from each parameter's conditional probability density functions. In a MCMC method, the authors employ the methodology combined a Gibbs sampler and a Metropolis-Hastings algorithm (Chib and Greenberg, 1995) (MH algolithm).

4. EMPIRICAL STUDY

4.1 Outline of visual inspection data

The Bayesian estimation for the multi-stage mixed Markov deterioration hazard model is carried out with the actual visual inspection data for expansion joints of expressway. The visual inspection results of the expansion joints are given four classifications. Table 1 shows specific valuation standards. When expansion joint's rating are classified in four steps, the hazard rate of rating 1, 2 and 3 (excluding rating 4) can be defined in the multi-stage mixed Markov deterioration hazard model. In the following description, the 1st stage group is defined as line, and the 2nd stage group is defined as each expansion joint.

The daily inspection for expansion joint can be divided into two kinds of inspection method: (1) the inspection viewing above position on the road such as the direct inspection driving the inspection vehicles and the tap test on the expressway with traffic control, and (2) the inspection viewing from below the expressway using binoculars targeting a leakage of water or an extraordinary noise. In the inspection on the road, the deterioration condition can be directly observed, but the frequency of inspection is low due to the traffic control. On the other hand, the inspection data from below the road without the traffic control is abundant. In this paper, the authors employed the inspection data from below the road to estimate the multi-stage mixed Markov deterioration hazard model because of the large quantity of the inspection data. Table 2 shows data specifications. The 10,689 expansion joint used in this paper are installed in 21 lines, and 32,273 samples can be obtained to estimate the multi-stage mixed Markov deterioration hazard model.

Table1 Description of condition state

Rating	Description of condition
1	Except the following conditions.
2	The case that it is necessary to observe condition of damage.
3	There is malfunction and it is necessary to take measures.
4	There is marked malfunction and it is necessary to take measures immediately considering safety.

The authors applied the multi-stage mixed Markov deterioration hazard model to the above mentioned visual inspection data of expansion joints, and estimated the unknown parameter vector $\boldsymbol{\beta}$, the heterogeneity parameter vector $\boldsymbol{\varepsilon}$ of the lines and the heterogeneity parameter vector $\boldsymbol{\varepsilon}'$ of the expansion joints. In Table 2, the average of the inspection intervals and the sample breakdown focusing on the prior and posterior ratings are shown. Based on previous study (Murakami et al., 2006), as the characteristic variable, the authors consider 14 items: (1) joint type, (2) traffic volume, (3) heavy traffic volume, (4) surface layer type, (5) grade, (6) radius of curvature, (7) acclivity, (8) road width, (9) traffic lane, (10) up and down line, (11) slab type,

Table 2 Data specifications

No. of lines			21 lines (line A to U)			
No. of expansion joints			10,689			
No. of samples			32,273			
Average of inspection intervals			2.846 years			
Sample breakdown			Posterior rating			
			1	2	3	4
	Prior rating	1	6,385	1,054	6,044	1,245
		2	-	920	985	211
		3	-	-	10,820	1,904
		4	-	-	-	2,675

Table 3 Estimated results of unknown parameters

Posterior distribution statistic	Rating i	Constant term $\beta_{i,0}$	Type of joint $\beta_{i,1}$	Type of surface layer pavement $\beta_{i,2}$	Heavy traffic volume $\beta_{i,3}$
Expected value (Geweke test statistic)	1	-1.121 (-0.788)	-0.560 (0.376)	0.184 (0.192)	0.744 (0.115)
Expected value (Geweke test statistic)	2	-0.062 (-0.382)	-0.272 (0.188)	0.406 (0.110)	-
Expected value (Geweke test statistic)	3	-2.806 (-0.191)	-0.133 (0.069)	0.270 (0.190)	0.337 (-0.109)

(12) declivity, (13) pavement area and (14) length of bridge. From these candidates, through the comparison of Geweke test statistic (Geweke, 1996) and AIC (deLeeuw, 1992), joint type, surface layer type and heavy traffic volume are finally selected as the characteristic variables in this paper. Note that the joint type and heavy traffic volume are considered by (Murakami et al., 2006) as a factor influencing the deterioration of expansion joints.

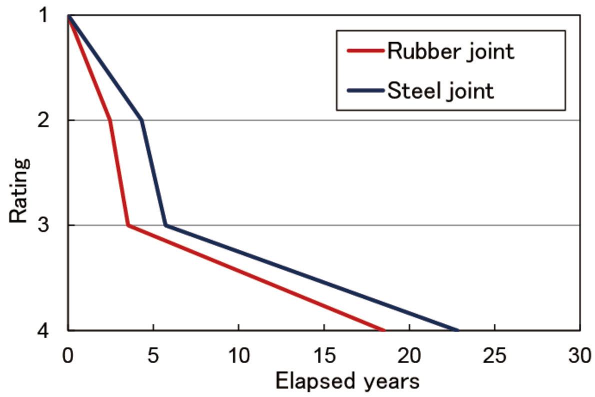
4.2 Estimation of expected deterioration paths

By employing the Bayesian estimation for the multi-stage mixed Markov deterioration hazard model, all unknown parameters included in the model can be simultaneously estimated. This chapter **4.2** mentions the change of expected deterioration paths and life expectancy due to a difference of characteristic variables. Chapter **4.3** and **4.4** describe heterogeneity parameters and life expectancy focused on each assessment unit.

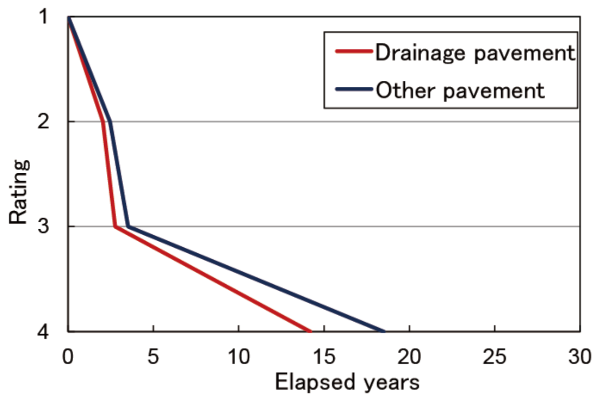
In the estimation of unknown parameter vector $\boldsymbol{\beta}$, three kinds of characteristic variable (joint type, surface layer type and heavy traffic volume) can be selected. Regarding the joint type, a dummy variable was established, the dummy variable is 1 when a joint type is steel or simplified steel, and the dummy variable is 0 when a joint type is rubber. In addition, regarding the surface layer type, another dummy variable was established, the dummy variable is 1 when a surface layer type is a drainage pavement, and the dummy variable is 0 when a surface layer type is excluding a drainage pavement (mainly dense graded asphalt concrete). The heavy traffic volume was normalized so that the maximum heavy traffic volume is 1. Estimated unknown parameter vector $\boldsymbol{\beta}$ is shown in Table 3. In 11 kinds of unknown parameter (excluding $\beta_{2,3}$), an absolute value of Geweke test statistic is lower than 1.96, and a sign condition is satisfied. A difference of the characteristic variable can be shown in Figure 2 as expected deterioration paths. Figure 2 (a) focuses on a joint type, (b) focuses on a surface layer type, and (c) focuses on a heavy traffic volume. Regarding the joint type, the life expectancy of a rubber joint is 18.5 years, and the life expectancy of a steel joint is 22.8 years. Figure 2 (a) can quantitatively show a rubber joint deteriorates fast than a steel joint. Regarding the surface layer type, a joint with drainage pavement deteriorates fast than it with other pavement. Regarding the heavy traffic volume, the larger traffic volume becomes, the faster a joint deteriorates.

4.3 Heterogeneity of lines

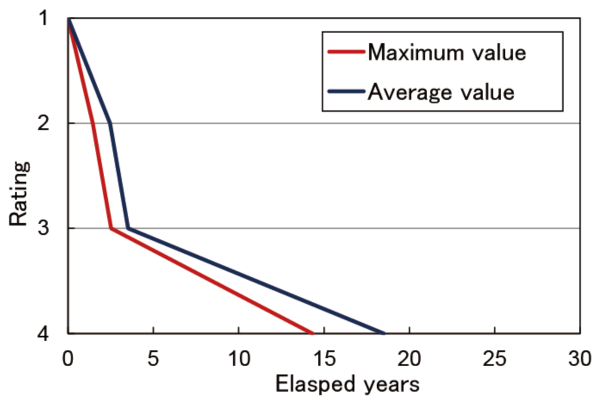
Heterogeneity parameter of the 1st stage group was set in each line. Figure 3 shows heterogeneity parameters of all 21 lines. The heterogeneity parameters of line vary from 0.347 to 2.017, and their average is 1.012. As mentioned in equation 5, the heterogeneity parameter influences the standard hazard rate as the product. This is why the difference among life expectancies of lines under the same characteristic variable condition can be quantitatively obtained by comparison of heterogeneity parameters. For example, in a deterioration rate's comparison between the line E with minimum heterogeneity parameter 0.347 and the line B with maximum parameter 2.017, the joint in line B deteriorates 2.017/0.347=5.991 times as fast as the joint in line E. Thus, by a comparison of only heterogeneity parameter, the relative evaluation of life expectancy becomes possible. For this reason, the heterogeneity parameter is very important index to compare life expectancies.



(a) Focused on joint type



(b) Focused on surface layer type



(c) Focused on heavy traffic volume

Figure 2 Changes of expected deterioration paths due to differences of characteristic variables

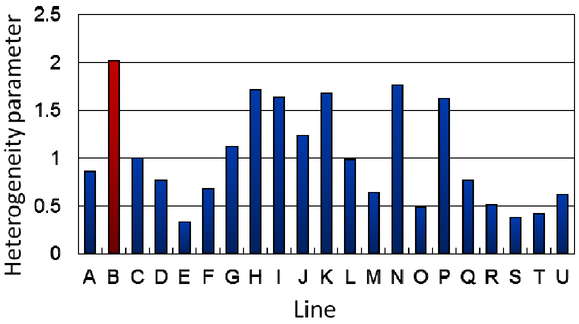


Figure 3 Estimated heterogeneity parameter of 21 lines

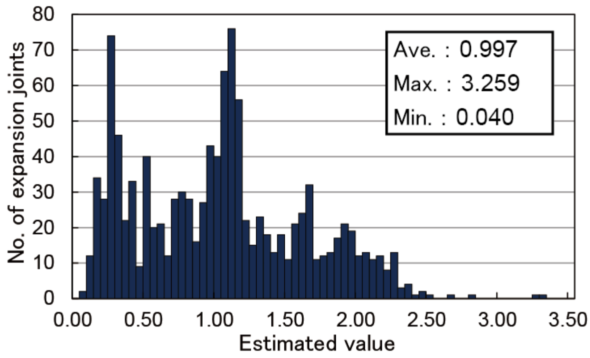


Figure 5 Distribution of 1,135 expansion joints' heterogeneity parameters in line B

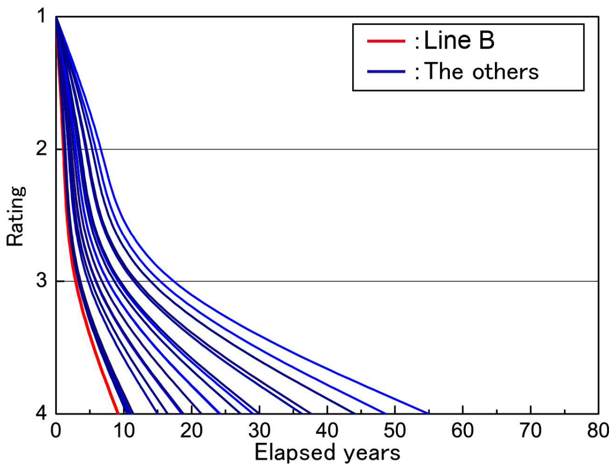


Figure 4 Expected deterioration paths considering heterogeneity of lines

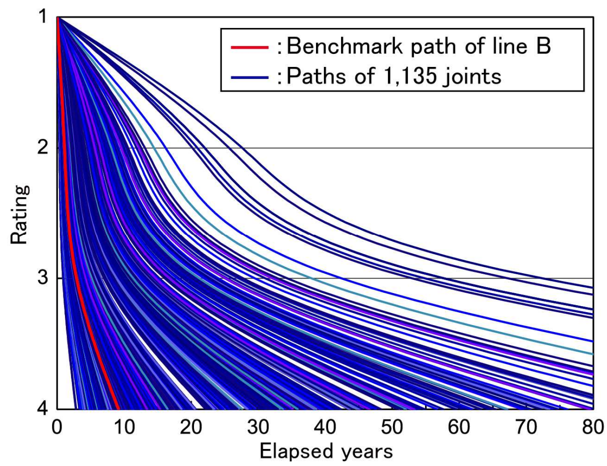


Figure 6 Expected deterioration paths of 1,135 joints in line B

In order to visually confirm the change of life expectancy, expected deterioration paths of 21 lines is shown in Figure 4. The life expectancy of line varies from about 10 years to 60 years. In Figure 4, as concrete characteristic variables, a joint type is the rubber joint, a surface layer type is the other pavement, and a normalized heavy traffic volume is the average value 0.294. The heterogeneity parameter set in each line can be utilized as benchmark of each line. The red curve in Figure 3 and 4 is line B. Line B deteriorates fast than the other lines. As detailed description in Section 4.5, to line B, the authors attempt extracting intensively monitored members.

4.4 Heterogeneity of expansion joints

Let us evaluate the heterogeneity of each expansion joint installed in line B. Figure 5 shows the distribution of 1,135 heterogeneity parameters in line B which are parameters of the 2nd stage group. The average value of the distribution is 0.997. The difference among deterioration rates of 1,135 joints can be expressed as life expectancies and expected deterioration paths. Figure 6 shows expected deterioration paths of 1,135 joints. As the heterogeneity parameter of the 1st stage group, the estimated heterogeneity parameter 2.017 of line B is employed, and the characteristic variables are same as Figure4. In Figure 6, the red curve indicates the benchmark path of all joints in line B which is same with the expected deterioration path of line B in Figure 4. In the case of consideration of budgetary allocation and the like in each line, the deterioration prediction result of targeting the 1st stage group is effective. On the other hand, it is necessary to predict the deterioration process in detail when the intensive monitored member is extracted in a specific line.

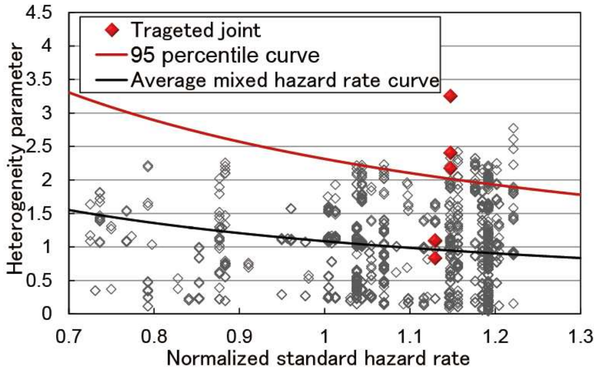


Figure 7 Relative evaluation of hazard rates

4.5 Extraction of intensively monitored members

By using the standard hazard rate and the heterogeneity parameter of each extraction joint, the intensive monitored member can be extracted from expansion joints installed in line B. Figure 7 shows the relation between rating 3's standard hazard rates of expansion joints in line B and heterogeneity parameters of each joint. The average value of the standard hazard rates is $AVE(\tilde{\lambda}_3^{s_{B,m}}) = 0.080$. The horizontal axis in Figure 7 indicates normalized standard hazard rates $\tilde{\lambda}_3^{s_{B,m}} / AVE(\tilde{\lambda}_3^{s_{B,m}})$. The mixed hazard rate of each expansion joint in line B is primarily defined as the product of a standard hazard rate, a heterogeneity parameter of each line and a heterogeneity parameter of each joint. In this section, in order to extract intensive monitored joints from a set of expansion joints in one line (line B), the heterogeneity parameter of line B is normalized as 1, and the analysis is conducted focusing on standard hazard rates and heterogeneity parameters of each joint. Since the average value of heterogeneity parameters of each joint is $AVE(\hat{\varepsilon}_{m|B}) = 0.997$, the average mixed hazard rate can be defined as $AVE(\tilde{\lambda}_3^{s_{B,m}}) * AVE(\hat{\varepsilon}_{m|B}) = 0.080 * 0.997 = 0.079$. The black curve indicates the average mixed hazard rate curve that a product of a standard hazard rate and a heterogeneity parameter of each joint becomes 0.079. It is indicated in Figure 7 that the mixed hazard rates of the expansion joints which lie above the average mixed hazard rate curve is larger than the average value (these joints deteriorate faster than the average), and the expansion joints which lie below the average mixed hazard rate curve deteriorate later than the average. In addition, Figure 7 shows the 95 percentile curve which indicates a product of a standard hazard rate and heterogeneity parameter of each joint. The intensively monitored members are the joints which lie above the 95 percentile curve. In this study, six expansion joints marked with a red color were extracted as the intensively monitored joint to practically install monitoring sensors. In Figure 7, the control level is set at 95% as one example. Needless to say, the control level should be determined by an administrator considering a number of targeted members, a budget and so on.

5. CONCLUSION

In this paper, the authors proposed the multi-stage mixed Markov hazard model and its multi hierarchical Bayesian estimation method in order to relatively evaluate deterioration rates corresponding to the decision making level. The empirical study targeting extension joints in viaducts of expressway was carried out. In the empirical study, first, deterioration rates of each line (the 1st stage group) were relatively evaluated. Second, the authors conducted the relative evaluation among deterioration rates of each joint (the 2nd stage group) installed in line B deteriorating faster than the other lines. Last, concrete joints monitored intensively were extracted to install monitoring sensors. Hence, it becomes possible to extract intensively monitored members with the systematized scheme utilizing the visual inspection data obtained in the present inspection system.

However, the authors have not discussed several points, which will be considered as topics for extending this study in the future.

- The monitoring system for extracted expansion joints as an intensively monitored member and the analytical method for the obtained monitoring data have not been discussed in this paper. It is necessary to construct the statistical algorithm to detect malfunction utilizing the continuously obtained data over a long period.
- The correlation between two kinds of inspection for expansion joints has not discussed in this paper. It is necessary to develop the simultaneous decision model of hazard rates or heterogeneity parameters by utilizing a copula function or the like.

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