ESTIMATION OF SPATIAL DISTRIBUTION OF ROAD ILLUMINANCE IN TUNNEL

Rentaro HAYASHI*, Satoshi HIRAKAWA**,
Kengo OBAMA* and Kiyoyuki KAITO*
Osaka University*,
West Nippon Expressway Company Limited**

***ABSTRACT* :** Fault of lamps in tunnel partially reduces road illuminance. Therefore, it becomes difficult to drive comfortably and safely. Thus, it is important for road administrators to quantitatively estimate fault probability of lamps and its effect. In this paper, the final target is to evaluate the risk of the reduction of road illuminance, caused by fault of lamps. Accordingly, the statistical methodology is constructed to analyze the influence and range of road illuminance the lamps have. Concretely, the relation between lamps and road illuminance is formulized as a spatial distribution of illuminance model, which takes into account of spatial interaction, and Bayesian estimation is proposed to estimate the unknown parameter and autocorrelation parameter of a spatial distribution of illuminance model. Finally, the validity of the proposed methodology is vertified through an application study using actual inspection data of a tunnel.

Key Words : *fault of tunnel lamps, spatial autoregressive model, Bayesian estimation*

1. INTRODUCTION

Tunnel lamps maintain illuminance in tunnel and give visual information such as cognition of a falling object and visual guidance to improve traveling safety. As present, lamps are installed based on mean road luminance and luminance evenness. On the other hand, adopting total revealing power to install lamps is under consideration[1]. Total revealing power is calculated by using a reflectance distribution and critical reflectance distribution of a variety of fallen objects considered as the multimodality of probability distributions. It is necessary to calculate critical reflectance for evaluating road illuminance. In this paper, illuminance is defined as follows: it is plane brightness which a light source supplies.

Fault of lamps in tunnel partially reduces road illuminance. Therefore, it becomes difficult to drive comfortably and safely. Thus, it is important for road administrators to quantitatively estimate fault probability of lamps and its effect. To evaluate the risk of fault of lamps respectively, a Weibull hazard model was formulated to express a deterioration process of lamps[2]. Moreover, the methodology to evaluate expected deterioration path and life expectancy was proposed[3]. On the other hand, there is the risk of the reduction of road illuminance caused by fault of lamps. However, as long as we know, there is no case that this risk was estimated quantitatively. Therefore, the statistical methodology is required to evaluate the risk by analyzing the illuminance which

Fig. 1 Illuminance measurement vehicle

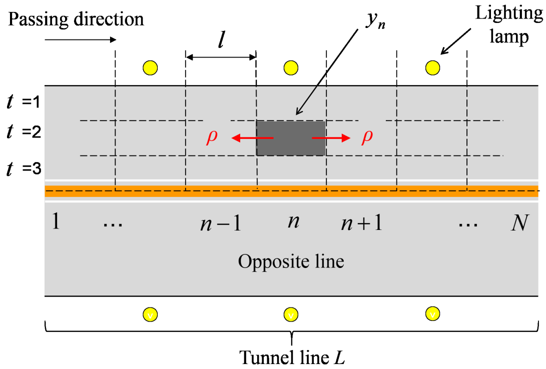

Fig. 2 Lighting environment in tunnel（case in T=1）

lamps supply to road surface and by expressing road illuminance precisely. Generally, road illuminance is measured by illuminance measurement vehicle. However, measured road illuminance depends on not only a single lamp but also an entire lamp installed in the tunnel. Therefore, with measured road illuminance, it is not easy to identify scalar and the range of road illuminance supplied by a single lamp installed in the tunnel. This relation with spatial interaction is expressed by spatial autoregressive model. In spatial econometrics, spatial autoregressive model is introduced[4]. The model expresses autocorrelation between explained variables by extending interrelation of time series in the direction of space. However, there is no case applied to the infrastructure.

In this paper, the statistical methodology is constructed to analyze the influence and range of road illuminance lamps supply. Concretely, the relation between lamps and road illuminance is formulated as a spatial distribution of illuminance model which considers spatial interaction. Moreover, Bayesian estimation is proposed to estimate unknown parameters and autocorrelation parameters of a spatial distribution of illuminance model. The paper is concluded by illustrating the application examples of the proposed methodology to the data set in the real field.

2. MODELING OF DISTRIBUTION OF ILLUMINANCE

2.1 Assumption of modeling

There are two kinds of tunnel lightings, entrance lighting and basic lighting. Entrance lighting supplies illuminance in the daytime and it's lamps are installed near the entrance to reduce the difference in the brightness between the inside and outside of a tunnel in the daytime. On the other hand, basic lighting supplies illuminance all the day and it's lamps are installed in tunnel at equal intervals. In this paper, premised the night lighting environment, it is supposed that only basic lighting in tunnel is turend on. Road illuminance is measured by illuminance measurement vehicle at regular intervals. Three measuring devices are installed in the front of illuminance measurement vehicle like **Figure–1**. Illuminance measurement vehicle continuously measures each illuminance such as left, center and right side of the width of road direction.

The length of a tunnel L is divided by N sections which unit length is l like **Figure–2**. The road illuminance y_n in a section n (range of n is $1, \cdots, N$ from an entrance to an exit) means horizontal illuminance measured by illuminance measurement vehicle in section n. However, three illuminances are measured to one lane by illuminance measurement vehicle. Thus, one lane is divided by three areas. The range of each area t is $1, \cdots, 3T$ like **Figure–2**. T is the number of lanes. We focus on an area t and analyze horizontal illuminance supplied to section n by lamps. Moreover, subscript about an area t is omitted to express simply. Horizontal illuminance supplied by lamps is expressed simply: (1) Horizontal illuminance of a section is supplied by only lamp of the section. (2) Lamp supplies the illuminance decreased in the ratio of uniformity to the section the lamp doesn't exist. (3) The reduction rate is exponential to length. Therefore, illuminence of section n (y_n) is defined by neighboring illuminance (y_{n-1} and y_{n+1}), existence of lamp of section n and the reduction rate of illuminance. Furthermore, illuminance supplied by each lamp in tunnel is equal and the difference in brightness occurred by deterioration and uncleanliness of lamps is not concerned.

2.2 Spatial distribution of illuminance model

Relation between lamps and illuminance is expressed by a spatial autoregressive model. In this paper, a spatial distribution of illuminance model means a spatial autoregressive model. Illuminance of a section changes with the existence of a lamp, or kinds of lamp. Therefore, characteristic of section n is expressed as characteristic vector, $\boldsymbol{x}_n = (x_n^1, \cdots, x_n^M)$ by characteristic variable, $x_n^m (m = 1, \cdots, M)$. In this paper, characteristic variables of the model are kinds of lamp (entrance or emergency parking area) and kinds of lane (passing or opposite). For example, basic lighting of passing lane is expressed as $m = 1$ and lighting of opposite lane in emergency parking area is $m = 4$. The characteristic variable concerned by those qualitative parameters is expressed as follows with dummy variable.

$$x_n^m = \begin{cases} 1 & existence\ of \\ & characteristic\ lamp\ of\ m \\ 0 & no\ existence\ of \\ & characteristic\ lamp\ of\ m \end{cases} \quad (1)$$

Furthermore, the reduction rate of illuminance described in **2.1** is expressed by spatial autocorrelation parameter ρ. Thus, illuminance y_n is expressed by y_{n-1} and y_{n+1}.

$$y_n = \rho y_{n-1} + \rho y_{n+1} + \boldsymbol{x}_n \boldsymbol{\beta} + \varepsilon_n \quad (2)$$

In this paper, we assume that tunnel lighting system is symmetrical lighting system. For this reason, neighboring spatial autocorrelation parameters are equal. On the other hand, there are two tunnel lighting systems: pro beam lighting system and counter beam lighting system. Pro beam system makes a leading vehicle easy to visually recognize and counter beam system makes it possible to secure the high road luminance to driver. Both tunnel lighting systems are asymmetrical lighting system. In the case that tunnel lighting system is asymmetrical one, it is necessary to mind that neighboring spatial autocorrelation parameters are not equal. Moreover, $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_M)'$ are unknown parameters and $'$ expresses transposition operation. Furthermore, ε_n is probability error term and is subject to the one-dimensional normal distribution $\mathcal{N}(0, \sigma^2)$. Here, formula (2) can be expressed by a matrix.

$$\mathbf{Y} = \rho \mathbf{W} \mathbf{Y} + \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (3)$$

Here, $\mathbf{Y} = (y_1, \cdots, y_N)'$, $\mathbf{X} = (\boldsymbol{x}_1', \cdots, \boldsymbol{x}_N')'$, $\boldsymbol{\varepsilon} = (\varepsilon_1, \cdots, \varepsilon_N)'$, and element of spatial weight matrix $\mathbf{W}$, $w_{i,j} (i = 1, \cdots, N; j = 1, \cdots, N)$, is expressed as follows:

$$w_{i,j} = \begin{cases} 1 & i = j \pm 1 \\ 0 & i \neq j \pm 1 \end{cases} \quad (4)$$

Formula (3) is a typical spatial autoregressive model and the likelihood is calculated by using probability error term $\boldsymbol{\varepsilon}$ being subject to the one-dimensional normal distribution $\mathcal{N}(0, \sigma^2)$. Unknown parameters are expressed $\boldsymbol{\theta} = (\boldsymbol{\beta}, \rho, \sigma)$. The likelihood to $\boldsymbol{\varepsilon}$ is expressed as follows:

$$\mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\varepsilon}) = \Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)^N \exp\Big(-\frac{\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}}{2\sigma^2}\Big) \quad (5)$$

Moreover, all data set obtained by illuminance measurement are expressed as $\bar{\boldsymbol{\Omega}} = (\bar{\mathbf{Y}}, \bar{\mathbf{X}})$ and formula (3) is expressed as follows:

$$\mathbf{A}\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta} = \boldsymbol{\varepsilon} \quad (6)$$

$$\mathbf{A} = \mathbf{I} - \rho\mathbf{W} \quad (7)$$

where $\mathbf{I}$ is identity matrix constituted by N columns and N rows. Changing valuable of formula (5) from $\boldsymbol{\varepsilon}$ to $\bar{\mathbf{Y}}$ by using formula (6) and (5), we can express the likelihood to $\bar{\mathbf{Y}}$ as follows:

$$\begin{aligned}&\mathcal{L}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}}) \\ &= \mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\varepsilon})\Big|\frac{\partial\boldsymbol{\varepsilon}}{\partial\bar{\mathbf{Y}}}\Big| = \mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\varepsilon})|\mathbf{A}| \\ &= \Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)^N \\ &\cdot \exp\Big\{-\frac{(\mathbf{A}\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta})'(\mathbf{A}\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta})}{2\sigma^2}\Big\}|\mathbf{A}|\end{aligned} \quad (8)$$

3. ESTIMATION OF SPATIAL DISTRIBUTION OF ILLUMINANCE MODEL

3.1 Summary of Bayesian estimation

There are some cases that spatial autoregressive model is estimated by maximum likelihood method. However, in many cases of actual asset management, only data of a limited quantity can be acquired. When a spatial autoregressive model is estimated by maximum likelihood method based on data of a limited quantity, systematic bias occur to estimate because maximum likelihood estimate is not satisfied with unbiasedness. In this paper, a spatial distribution of illuminance model is estimated by Bayesian method with MCMC (Markov Chain Monte Marlo) method. Bayesian estimation can be estimated with comparatively sufficient accuracy to utilize prior information, even when there are few samples. Moreover, Bayesian estimation is useful for risk management because it is possible to examine the credible interval of estimate.

In general Bayesian estimation, posterior probability distribution of parameter is estimated by using prior probability distribution of parameter and by using the likelihood based on observation information. Here, it is assumed that unkown parameter vector $\boldsymbol{\theta}$ is random variable and it is subject to prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$. When measurement data is obtained, simultaneous posterior probability density function of unkown parameter vector $\boldsymbol{\theta}$ is expressed by Bayes' theorem. Thus, $\boldsymbol{\pi}(\boldsymbol{\theta})$ is expressed with prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ and likelihood$\mathcal{L}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}})$ as follows:

$$\boldsymbol{\pi}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}}) \propto \mathcal{L}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}})\boldsymbol{\pi}(\boldsymbol{\theta}) \quad (9)$$

3.2 Formulation of posterior probability distribution

Prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ can be developed.

$$\boldsymbol{\pi}(\boldsymbol{\theta}) = \boldsymbol{\pi}(\boldsymbol{\beta}, \rho, \sigma) = \pi(\boldsymbol{\beta})\pi(\rho)\pi(\sigma) \quad (10)$$

Prior probability density function of unknown parameter vector of formula (10) is set up as follows. First, $\pi(\boldsymbol{\beta})$, prior probability density function of $\boldsymbol{\beta}$, is multidimensional normal distribution; $\boldsymbol{\beta} \sim \mathcal{N}(\boldsymbol{b}_0, \Sigma_{\boldsymbol{b}_0})$. $\pi(\rho)$, prior probability density function of ρ, is one dimensional normal distribution; $\rho \sim \mathcal{N}(\rho_0, P_0)$. prior probability density function of probability error term $\pi(\sigma)$ is inverse-gamma distribution; $\sigma \sim \mathcal{IG}(n_0/2, S_0/2)$. Lower right subscript 0 expresses a hyperparameter. From the above, simultaneous posterior probability density function can be expressed by formula (8), (9) and (10).

$$\begin{aligned}&\boldsymbol{\pi}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}}) \\ &\propto (\sigma^2)^{(-\frac{N}{2})}\end{aligned}$$

$$\cdot \exp\Big\{-\frac{(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})'(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})}{2\sigma^2}\Big\}|\mathbf{A}|$$
$$\cdot \exp\Big\{-\frac{1}{2}(\boldsymbol{\beta}-\boldsymbol{b}_0)'\Sigma_{\boldsymbol{b}_0}^{-1}(\boldsymbol{\beta}-\boldsymbol{b}_0)\Big\}$$
$$\cdot \exp\Big\{-\frac{1}{2P_0}(\rho-\rho_0)^2\Big\}$$
$$\cdot(\sigma^2)^{(-\frac{n_0}{2}+1)}\exp\Big\{-\frac{S_0}{2\sigma^2}\Big\} \tag{11}$$

3.3 Estimation of simultaneous posterior probability density function

To calculate simultaneous posterior probability density function of spatial autoregressive distribution model $\boldsymbol{\psi}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}})$, Gibbs sampling using conditional posterior probability density function is applied. Simultaneous posterior probability density function is calculated, by classifying unknown parameter vector $\boldsymbol{\theta}$ to each parameter $\boldsymbol{\beta}, \rho, \sigma$, and nextly, by repeating random sampling based on conditional posterior probability density function with other parameters as known values.

Simultaneous posterior probability density function is calculated by using conditional posterior probability density function of each parameter. $\boldsymbol{\psi}(\boldsymbol{\beta}|\rho,\sigma,\bar{\boldsymbol{\Omega}})$, conditional posterior probability density function of $\boldsymbol{\beta}$ with parameter ρ and σ as known values, can be expressed as

$$\begin{aligned}&\boldsymbol{\psi}(\boldsymbol{\beta}|\rho,\sigma,\bar{\boldsymbol{\Omega}})\\ &\propto \exp\Big\{-\frac{1}{2}(\boldsymbol{\beta}-\boldsymbol{b}_1)'\Sigma_1^{-1}(\boldsymbol{\beta}-\boldsymbol{b}_1)\Big\}\\ &\quad \boldsymbol{b}_1=\Sigma_1(\Sigma_0^{-1}\boldsymbol{b}_0+\sigma^{-2}\bar{\mathbf{X}}'\mathbf{A}\bar{\mathbf{Y}})\\ &\quad \Sigma_1^{-1}=\Sigma_0^{-1}+\sigma^{-2}\bar{\mathbf{X}}'\bar{\mathbf{X}}\end{aligned} \tag{12}$$

In other words, $\boldsymbol{\psi}(\boldsymbol{\beta}|\rho,\sigma,\bar{\boldsymbol{\Omega}})$ is multidimensional normal distribution $\mathcal{N}(\boldsymbol{b}_1,\Sigma_1)$. Furthermore, conditional posterior probability density function of ρ, with parameter $\boldsymbol{\beta}$ and σ as known values, is expressed as

$$\boldsymbol{\psi}(\rho|\boldsymbol{\beta},\sigma,\bar{\boldsymbol{\Omega}})\propto|\mathbf{A}|\cdot\exp\Big\{-\frac{1}{2P_0}(\rho-\rho_0)^2 -\frac{(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})'(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})}{2\sigma^2}\Big\} \tag{13}$$

Moreover, conditional posterior probability density function of σ, with parameter $\boldsymbol{\beta}$ and ρ as known values, can be expressed as

$$\begin{aligned}&\boldsymbol{\psi}(\sigma|\boldsymbol{\beta},\rho,\bar{\boldsymbol{\Omega}})\\ &\propto(\sigma^2)^{(-\frac{n_1}{2}+1)}\cdot\exp\Big\{-\frac{S_1}{2\sigma^2}\Big\}\\ &\quad n_1=n_0+N\\ &\quad S_1=S_0+(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})'(\mathbf{A}\bar{\mathbf{Y}}-\bar{\mathbf{X}}\boldsymbol{\beta})\end{aligned} \tag{14}$$

In other words, $\boldsymbol{\psi}(\sigma|\boldsymbol{\beta},\rho,\bar{\boldsymbol{\Omega}})$ is inverse-gamma distribution $\mathcal{IG}(n_1/2, S_1/2)$. simultaneous posterior probability density function of formula (11) is calculated by those conditional posterior probability density functions. Concrete presumed steps are shown below and in **Figure–3**.

step1
: Parameters of prior probability distribution $\boldsymbol{b}_0, \Sigma_{\boldsymbol{b}_0}, \rho_0, P_0, n_0/2, S_0/2$ are set up arbitrary. Furthermore, $\boldsymbol{\theta}^{(0)}=(\boldsymbol{\beta}^{(0)},\rho^{(0)},\sigma^{(0)})$, initial values of unknown parameters $\boldsymbol{\theta}=[\boldsymbol{\beta},\rho,\sigma]$, are set up arbitrary. The more the number of samplings increases, the more the influence of an initial value fades. The initial value is unrelated to the sampling after converging on a steady state.

step2-1
: $\boldsymbol{\beta}^{(n)}$, partial vector of unknown parameter of sampling number n, is sampled at random from $\boldsymbol{\pi}(\boldsymbol{\beta}|\rho^{(n-1)},\sigma^{(n-1)},\bar{\boldsymbol{\Omega}})$.

step2-2
: $\rho^{(n)}$, partial vector of unknown parameter of sampling number n, is sampled at random from $\boldsymbol{\pi}(\rho|\boldsymbol{\beta}^{(n)},\sigma^{(n-1)},\bar{\boldsymbol{\Omega}})$.

step2-3
: $\sigma^{(n)}$, partial vector of unknown parameter of sampling number n, is sampled at random from $\boldsymbol{\pi}(\sigma|\boldsymbol{\beta}^{(n)},\rho^{(n)},\bar{\boldsymbol{\Omega}})$.

step3
: If $n>\underline{n}$ to sufficiently large $\underline{n}$, $\boldsymbol{\theta}^{(n)}=(\boldsymbol{\beta}^{(n)},\rho^{(n)},\sigma^{(n)})$ is recorded.

step4
: If $n=\overline{n}$, the calculation is ended. If $n<\overline{n}$, then $n=n+1$ so returns to **step2**.

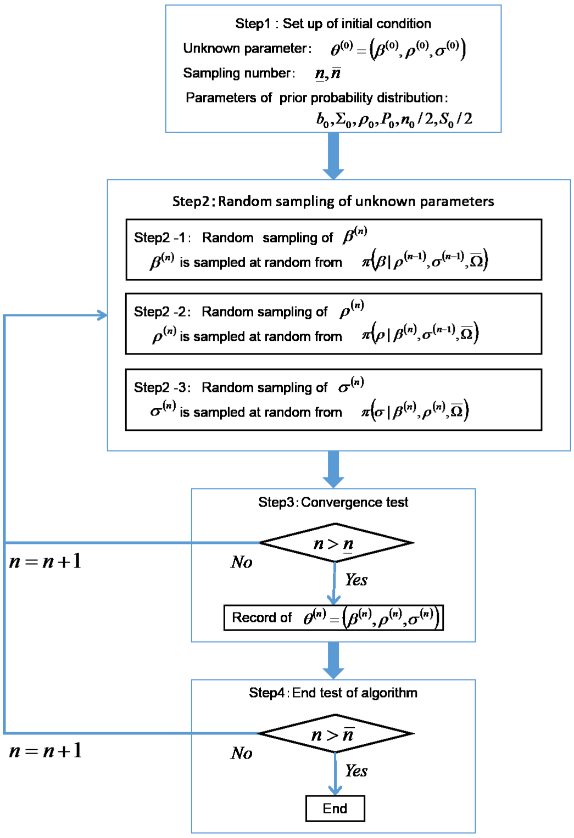

Fig. 3 Estimation flow of spatial illuminance distribution model

If this Markov Chain has reached the steady state, sampling of $\boldsymbol{\theta}(n = \underline{n} + 1, \underline{n} + 2, \cdots, \overline{n})$ by Gibbs sampling is equal to that of $\boldsymbol{\pi}(\boldsymbol{\theta}|\bar{\boldsymbol{\Omega}})$ shown in formula (9). Accordingly, with these sample $\boldsymbol{\theta}(n = \underline{n} + 1, \underline{n} + 2, \cdots, \overline{n})$ from Gibbs sampling, it is able to calclate the statistic of simultaneous posterior probability density function;$\boldsymbol{\theta} = [\boldsymbol{\beta}, \rho, \sigma]$. However, there are some parameters which cannot be sampled at random directly from conditional posterior probability density functions.

3.4 Sampling from conditional posterior probability function

Even if partial vector of unknown parameter cannot be sampled directly from conditional posterior probability density function, it is possible to obtain the sample from conditional posterior probability density function by using random walk MH (Metropolis Hastings) algorithm. Concretely, random walk MH algorithm is applied to sampling from $\boldsymbol{\psi}(\rho|\boldsymbol{\beta}^{(n)}, \sigma^{(n-1)}, \bar{\boldsymbol{\Omega}})$. MH algorithm is the method that partial vector of unknown parameter is sampled from proposal distribution which approximates target distribution and the difference between proposal distribution and target distribution is corrected. If those operations are repeated, the sample sampled in the number of times of a repetition large enough can be regarded as the sample sampled at random from target distribution. Here, target distribution is expressed as $\boldsymbol{\psi}(\dot{\theta}|\dot{\bar{\boldsymbol{\Omega}}})$ and probability density function of proposal distribution is expressed as $\boldsymbol{q}(\dot{\theta}'|\dot{\theta}^{(n-1)})$. The n-th sampling from proposal distribution generates $\dot{\theta}'$ following $\boldsymbol{q}(\dot{\theta}'|\dot{\theta}^{(n-1)})$ as the candidate parameter from posterior probability distribution. The candidate $\dot{\theta}'$ is not one generated from target distribution. Thus, to current the difference, the probability that the candidate $\dot{\theta}'$ is adopted is defined as follows:

$$\alpha(\dot{\theta}'|\dot{\theta}^{(n-1)}) = \min\left[\frac{\boldsymbol{\psi}(\dot{\theta}'|\dot{\bar{\boldsymbol{\Omega}}})\boldsymbol{q}(\dot{\theta}^{(n-1)}|\dot{\theta}')}{\boldsymbol{\psi}(\dot{\theta}^{(n-1)}|\dot{\bar{\boldsymbol{\Omega}}})\boldsymbol{q}(\dot{\theta}'|\dot{\theta}^{(n-1)})}\right] \tag{15}$$

If adopted, $\dot{\theta}^{(n)} = \dot{\theta}'$. If rejected, $\dot{\theta}^{(n)} = \dot{\theta}^{(n-1)}$. In this paper, it is sampled from $\boldsymbol{\psi}(\rho|\boldsymbol{\beta}^{(n)}, \sigma^{(n-1)}, \bar{\boldsymbol{\Omega}})$ with random walk MH method. The n-th candidate is generated by random walk such as below.

$$\rho' = \rho^{(n-1)} + \mathcal{N}(0, P) \tag{16}$$

Since probability density q is symmetrical about $(\rho', \rho^{(n-1)})$, the candidate ρ' is adopted by the probability such as below.

$$\alpha_\rho(\rho'|\rho^{(n-1)}) = \min\left[\frac{\boldsymbol{\psi}(\rho'|\boldsymbol{\beta}^{(n)}, \sigma^{(n-1)}, \bar{\boldsymbol{\Omega}})}{\boldsymbol{\psi}(\rho^{(n-1)}|\boldsymbol{\beta}^{(n)}, \sigma^{(n-1)}, \bar{\boldsymbol{\Omega}})}, 1\right] \tag{17}$$

In actual numerical computation, it generated uniform random numbers u. u is subject to uniform distribution between $[0, 1]$. Nextly, sample are determined according to the following rules:

$$\rho^{(n)} = \begin{cases} \rho^{(n-1)} & u > \alpha_\rho \\ \rho' & u \leq \alpha_\rho \end{cases} \quad (18)$$

The above is equivalent to **Step2-2** of the Gibbs sampling shown by **3.3**.

4. EMPIRICAL STUDY

4.1 Summary of applied case

In this study, the subject tunnel has a road with double lane and symmetrical lighting system. The illuminance data is acquired at night in March, 2011 by illuminance measurement vehicle. Furthermore, the position of lamps and kinds of lamps are obtained by design drawing. High pressure natrium lamps are installed in basic lighting. Fluorescent lamps are installed in emergency parking area. The length of tunnel is 2,794m and interval of lamps is 12.5m equally. The tunnel has three emergency parking areas and the interval of lamps in the area is 2.2m. The data measured by illuminance measurement vehicle are the average illuminance in every 1m. In this paper, section l is applied 6.25m and the illuminance of each section is applied the average illuminance in each section. Therefore, the number of section in each area is 448 sections to one lane. Moreover, since the number of lane T is 1 and the number of area is 3 in the tunnel, total number of section in each area is 896. The point of fault of lamps is inspected regularly. In this paper, fault of lamps are defined as the failure lamps discovered by inspection and the replaced lamps after the latest illuminance measurement. The point of fault of lamps is defined as where lamp is not installed. Based on the above data, a spatial distribution of illuminance model is estimated.

4.2 Estimated results of model

In this paper, characteristic variables of the model are determined by kinds of lane (passing lane or opposite lane) and kinds of lighting (basic lighting or lighting of emergency parking area). Characteristics m are expressed as

$$\begin{cases} m = 1 & passing\ lane,\ basic\ lighting \\ m = 2 & oppsite\ lane,\ basic\ lighting \\ m = 3 & passing\ lane,\ lighting\ of \\ & emergency\ parking\ area \\ m = 4 & oppsite\ lane,\ lighting\ of \\ & emergency\ parking\ area \end{cases} \quad (19)$$

Therefore, $\boldsymbol{\theta}$, unknown parameter vector of area t, is $\boldsymbol{\theta} = (\beta_1, \beta_2, \beta_3, \beta_4, \rho, \sigma)$.

As an example, estimated result of area 1($t = 1$) is shown in **Table–1**. **Table–1** shows Sample average value (called an "estimate" simply), 90% credible interval $(\underline{\theta}_k^\kappa, \overline{\theta}_k^\kappa)$, Geweke test statistics[5], log likelihood and AIC. The model which minimizes AIC is applied as the optimal model in the model of the combination of various characteristic variables. As a result, in this paper, all characteristic variables are applied. The estimate of parameter $\beta_m (m = 1, 2, 3, 4)$ expresses the value of illuminance which the lamp of m supplies to the section. For example, the estimate of β_1 expresses that the lamp of $m = 1$ supplies 44.04(lx) if the lamp installed passing lane in basic lighting turns on. Furthermore, the estimate of ρ expresses that the lamp supplies the adjoining section the illuminance decreased at a rate of 0.310. **Table–1** shows that the influence of β_3 on the tunnel greater than that of β_1. Three factors are mentioned about this. The 1st factor is the difference between kinds of lamp. The 2nd factor is influences of light distribution. Light distribution curves are difference between basic lighting and lighting of emergency parking area. Especially, light distribution in an emergency parking area is set up to make the lane of left-hand side mentiond as this exsample ($t = 1$) the bright-

Table 1 Estimated results of spatial illuminance distribution model （$t = 1$）

Posterior distribution statistic	Passing line basic lighting β_1	Opposite line basic lighting β_2	Passing line Emerge area β_3	Opposite line Emerge area β_4	Spatial autocorrelation ρ	Error term standard deviation σ
Expexted value	44.04	15.27	39.19	6.55	0.310	13.67
90%credible interval	(41.0, 47.0)	(12.3, 18.2)	(35.1, 43.2)	(2.53, 10.5)	(0.30, 0.32)	(13.1, 14.2)
Geweke test statistics	0.041	0.045	0.064	0.052	-0.002	0.004
Log likelihood	-3,692					
AIC	7,397					

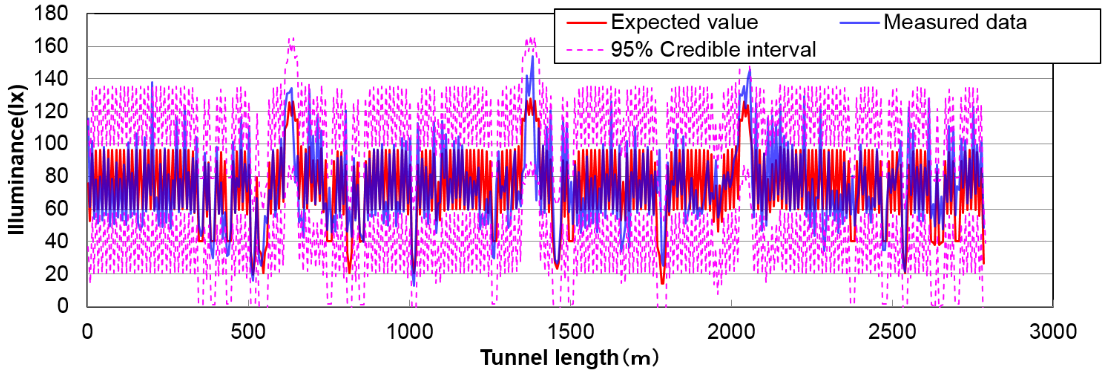

Fig. 4 Expected value and measured data of illuminance （$t = 1$）

est. The 3rd factor is a difference in the number of lamps installed in the one section. Since the installation interval is difference between basic lighting and lighting of emergency parking, the lamps in emergency parking is installed more densely than that of basic lighting. However, in this study, an explanatory variable is the existence of lamp to each section. Thus, difference in the number of lamps was not taken into consideration. The data and expected value of the model are shown in **Figure–4**. The target data is about the up line. The data is drawn by the blue line and expected value of the model is drawn by the red line. Moreover, 95% credible interval of the expected value of the model is drawn by the peach color dotted line in **Figure–4**. Thereby, **Figure–4** shows that the overall tendency of the data is expressed in 95 % credible interval of the expected value of the model. However, there are some section which is less than the lower limit of 95 % credible interval. The factor is a time lag on an illuminance measurement date and an inspection date discovered fault of lamps. In this paper, an illuminance measurement day does not accord with an inspection day. Thus, the existence of fault of lamp is not expressed accuracy. In order to raise estimation accuracy from now on, it should be considered that an inspection day of fault of lamps is equal to illuminance measurement day. For this reason, in order to manage illuminance in tunnel easily, not only is the expected value of the illuminance of the whole tunnel grasped but also the section which is less than the lower limit of a 95% credible interval is inspected individually is required. In any case, it is important to construct a systematic decision-making process based on quantitative data about tunnel lighting.

4.3 Estimation of the reduction of illuminance caused by fault of lamps

Since the influence which lamps has on illuminance is evaluated quantitatively, the spatial illuminance distribution model in this paper makes it possible to evaluate risk of the reduction of illuminance by fault of lamps. The case where fault of lamps occur in lamp of passing lane is considered. When illuminance is evaluated between estimate of 95% credible interval, the illuminance is reduced 41.0 (lx) to 47.0 (lx) at l=6.25m. Moreover, **Figure–4** shows that 95% credible interval of parameter ρ is 0.30 to 0.32. Therefore, if fault of lamps occur in lamp of passing lane, the illuminance of adjoining section is reduced 12.3 (lx) to 15.0 (lx). Next, if fault of lamps occur in the lamps of two continuous sections, the illuminance is subject to both of influences by fault of lamp occuring in the section and the adjoining section. Thus, the illuminance reduced 53.3 (lx) to 62.0 (lx). From the above results, it is noted that the spatial illumination distribution model enabled quantitive evaluation of the reduction of illuminance. On the other hand, it is necessary to set up a short section and a peculiar characteristic variable in section in order to express spatial change of illuminance in tunnel.

4.4 A future subject

It was noted that the risk of the illumination by the fault of lamps can be evaluated by this model in **4.(3)**. However, there is a problem that probability error term is too large. **Table–1** shows that it has an influence that is equal to other parameters in spite of probability error term. Two things can be considered as a factor of this. One is not taking into consideration about the difference in the brightness for every lamp in this model. It is necessary to develop the model which takes into consideration about the influence of a reduction of illuminance of each lamp by aged deterioration from now on. The other is a problem of the size of the interval of the section l. In the size of the present section, the difference in the number of the lamp installed in the one section cannot be taken into consideration. In this paper, the illuminance of each section is the average illuminance of section l. Since interval of the section l is large, there is a possibility of illuminance being smoothed and having underestimated the credible interval. From the above reasons, it is necessary to estimate by the shorter section and it is important to measure illuminance by the shorter section. However, when the section is shortened, there is some possibility of that the influence of light distribution becomes large. It is necessary for modeling to take light distribution into consideration. It is also important to continue recording a repair history besides at the tunnel lamps installation-time etc.

5. CONCLUSION

This paper attempts to propose methodology to develop a spatial distribution model and analyze the influence and range of road illuminance lamps supply in order to evaluate the risk of the reduction of road illuminance caused by fault of lamps. Moreover, this model is applied to the illuminance data of an actual tunnel. As a result, it was found that the reduction of illuminance by fault of lamps can be evaluated quantitatively. Furthermore, it pointed out that it was necessary to mind about a setup of the section.

Future deployment of the proposed methodology is as follows. In addtion to the subjects described by **4.(4)**, the following research tasks will be mentioned. Firstly, it is calculation of total revealing power using the illuminance distribution model proposed in this research. In examination of the present tunnel lighting system, an important index is critical reflectance of positive contrast and negative contrast to fallen object. Since vertical illuminance is necessary to calculate critical reflectance, this illuminance distribution will be enable to calculate vertical illuminance in consideration of uncertainty, and will be enable to construct the determination method of tunnel lighting system. Secondly, this model is applied to evaluation of the risk when fault of lamps occur continuously. It is important to develop evaluation model of the various risk such as the risk of fault of lamps expressed as Weibull hazard model and the risk of reduction of illuminance caused by dirt in tunnel.

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