THE OPTIMAL SCRAPPING AND MAINTENANCE MODEL OF INFRASTRUCTURE CONSIDERING DETERIORATION PROCESS

Taiki FUKUDA*, Kiyoshi KOBAYASHI**, Kiyoyuki KAITO***

Osaka University* Kyoto University** Osaka University***

ABSTRACT: In our country, entering the decrease period of the population, it is difficult to secure a source of revenue taken for maintenance infrastructures exist. In this circumstance, the demand for scrapping deteriorated infrastructures, and shrinking infrastructure network becomes to be argued. Infrastructures are durable, and can be used continuously as long as there are no problems in safety or economic performance. In the maintenance management to infrastructures, it becomes necessary to study the long-term scrapping and maintenance policy of infrastructures as the service level of the plant maintenance is controlled based on a long-term trend of infrastructures demand, and long-term scrapping infrastructures is studied. In this study, the authors propose a methodology for designing optimal scrapping and maintenance policies for infrastructures that are to be demolished due to the demand decline or the deterioration progression. In particular, The infrastructure that demand decreases surely in the future and the facilities abandonment is determinably determinately are assumed, and this study is focused on the three scrapping and maintenance policies: (1) to demolish infrastructures immediately, (2) to maintain them under the minimum maintenance, and (3) to repair them constantly. Then, the authors propose optimal scrapping and maintenance models for designing asset management policies considering the demand and deterioration of infrastructures. In addition, the effectiveness of the proposed optimal scrapping and maintenance models is investigated through a case study. And one of reasonable criterion for scrapping and maintenance of infrastructures is suggested.

KEYWORDS: optimal scrapping, maintenance & rehabilitation

1. INTRODUCTION

When long-term management policy is considered, the environmental characteristic of infrastructures becomes a very important problem. In this study, the management policy of infrastructures that demand decreases surely, and will be demolished determinately is considered. For infrastructures to be demolished determinately, three management policies, (1) to demolish infrastructures immediately, (2) to maintain them under the minimum maintenance, and (3) to repair them constantly can

be applied. The administrator selects a preferable management policy while considering the benefit stream in the future that the operation brings in.

Under the problem consciousness, in this study, the authors target infrastructures to be demolished in the future, and suggest the methodology to seek an optimal management policy that minimizes discount cash flow of expected net benefit. By the methodology proposes in this study, though it's a limited standpoint, it is thought that one reasonable judgment information can be suggested concerning infrastructures management policy. **2.** explains the

basic idea for this study. **3.** suggests the optimal scrapping and maintenance model. **4.** applies the model targeting a bridge.

2. THE BASIC IDEA FOR THIS STUDY

the management policy In this study, of infrastructures concerning the uncertainty of the deterioration process is considered. In this regard, as infrastructures management policy, three management policies are picked up (1) waste policy, (2) waste reservation policy, (3) repair policy. The abandonment policy discontinues the use of infrastructures at once, and waste them. When infrastructures are demolished, the waste cost is needed. When infrastructures are demolished, the waste cost is needed, but by reason of life of infrastructures is long, and deterioration progress of infrastructures is slow, it is not economic to demolish at once. Then, waste reservation policy is not to take major repair but to maintain them under the minimum maintenance, cleaning and road patrol, and decide the operation that an infrastructure is demolished at the appropriate time. Finally, repair policy works out day-to-day maintenance and major repair when deterioration and damage are found, and the soundness of infrastructure is recovered. In addition, in this study, the determinate demand change that infrastructures demand decreases monotonously is assumed.

Under the circumstances, the infrastructures management policy's being monotonously change with the time passage in order "Repair policy \rightarrow waste reservation policy \rightarrow waste policy", deciding the optimal policy switch timing becomes a problem.

3. OUTLINE OF MODEL

A certain case that the optimal maintenance and the repair policy of infrastructures are executed at the time of initial, and it arrives at t period become a consideration. At t period, The administrator selects

it from three management policies of the following: (1) to demolish infrastructures immediately (waste policy), (2) to maintain them under the minimum maintenance(waste reservation policy), and (3) to repair them constantly(repair policy). When the administrator adopt waste policy, the infrastructure is demolished at once by bearing waste cost C. Next, when the administrator adopt waste reservation policy, the operation that is a major repair that recovers the deterioration level is not implemented though the operation that is the regular minimum maintenance by bearing management cost is the implemented. When deterioration of infrastructures progresses along with the time passage, and it's soundness reaches application limits I, infrastructures are demolished. Because of uncertainty of deterioration progress, the time of waste of infrastructures is uncertain. Waste cost Cbecomes necessary to demolish infrastructures. waste cost includes restriction cost and dismantlement cost of the bridge. Finally, when the repair policy is implemented, the administrator continues maintenance and repair. Then, the repair rule is decided to maximize the value for period of the expected net benefit by infrastructures use considering the possibility of the infrastructure waste in the future. In the following, the authors consider time passes at the time of initial, and at the beginning of period of the *t* period, the administrator decides the management policy. Hereafter, in 3.2, the case that the administrator adopt waste policy is focused. Next in 3.3, the case that the administrator adopt waste reservation policy, and in 3.4, the case that the administrator adopt repair policy are focused. Finally, the optimal scrapping and maintenance model is formulated by **3.5**.

3.1 Waste policy

The authors assume that t is thought present time, the inspection is done at the beginning of period at the

time *t*, and it turned out that the soundness of infrastructures was *i*. The benefit of infrastructure will not be generated in the future because it is demolished at once when the administrator adopts "waste policy". Therefore, the net benefit at present $\Psi_t^a(i)$ as follows:

$$\Psi_t^a(i) = -C \tag{1}$$

Here, upper index a indicates that waste policy is adopted.

3.2 Waste reservation policy

When waste reservation policy is adopted, the repair operation that influences the deterioration process of facilities is not implemented though minimum maintenance operation of the road patrol and the cleaning, etc. is executed. The soundness of facilities decreases with the time passage, and soundness reaches application limits *I*, facilities are demolished. It is thought that the deterioration process of facilities changes according to the Markov chain^{1),2)}. Then, the transition of soundness between the time of two is expressed at the Markov transition probability. Markov transition probability assumes soundness h(t)=i measured at time t to be a donnee, and is defined as a transition probability with the condition that soundness $h(t+1)=j(j \ge i)$ occurs at the t+1. Therefore, as follows:

$$Prob[h(t+1) = j | h(t) = i] = \pi_{ii}$$
(2)

Such a transition probability is defined with soundness pair (i,j), the Markov transition probability procession can be defined.

$$\Pi = \begin{pmatrix} \pi_{11} & \cdots & \pi_{1I} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \pi_{II} \end{pmatrix}$$
(3)

At this time, because deterioration always progresses as long as there is no repair, $\pi_{ij}=0$ (i > j) is true. Additionally, from definition of the transition

$$\pi_{ij} \ge 0(i, j = 1, \dots, I)$$
probability, $\Sigma_{j=i} \pi_{ij} = 0(i > j)$

$$\sum_{j=i}^{I} \pi_{ij} = 1$$

$$(4)$$

The condition *I* is absorption in Markov chain, and $\pi_{II}=1$ is true. Markov transition probability is defined independently of a past deterioration history. Here, the authors think that as time passes at the initial time, the state that reaches the *t* period. The soundness of facilities at an applicable time is shown h(t) = i ($i=1, \dots, I$). Here, waste reservation policy is adopted for facilities of soundness h(t)=i at *t* period, and discount cash flow of expected net benefit that can be acquired when the optimal management policy (hereafter, discount cash flow of waste reservation policy) is adopted after the next term is indicated $\Psi_t^{b}(i)$. At first, when soundness of a facility is *I* at the *t* period, discount cash flow of waste reservation policy becomes as follows:

$$\Psi_t^b(i) = -C \tag{5}$$

Next, the case where the soundness at *t* period is *i* is considered. Here, when waste reservation policy is adopted in the condition whose soundness of facilities is h(t) = i at the beginning of period *t*, discount cash flow of waste reservation policy is shown as follows:

$$\Psi_{t}^{b}(i) = v(t) - c + \rho \sum_{j=i}^{l} \pi_{ij} V_{t+1}^{b}(j)$$
(6)

In this regard, ρ (0< ρ <1) is discount factor and in this study, ρ =0.96. Moreover, $V_{t+1}{}^{b}(j)$ is the maximum value of discount cash flow of expected net benefit that can be acquired by selecting the optimal management policy under the soundness *j* at period *t*.

Because the facilities convenience decreases monotonously, if waste reservation policy is adopted once, the repair policy is not adopted.

When the authors take notice waste reservation policy is selected and waste policy has never been selected before the period t, the multiple choice question of the optimal management policy at period t can be formulated as follows:

$$V_t^b(i) = \max\left\{\Psi_t^b(i), -C\right\}$$
(7)

By recurrently applying expression (7) and (8) at an arbitrary period, the waste reservation policy can be applied, and discount cash flow $\Psi_t^{b}(i)$ of waste reservation policy when the optimal management policy is applied after that can be defined. If benefit v(t) is a strong monotone decreasing function, as long as soundness *i* is the same, the following holds:

$$\Psi_t^b(i) > \Psi_{t+1}^b(i) > \cdots \tag{8}$$

On the other hand, $\Psi_t^a(i)$ takes definite value -*C* regardless of the value of *t* and *i*. Therefore, $t^*(i)$ that the following holds to arbitrary *i* exists.

$$\Psi^{b}_{t^{*}(i)-1}(i) > -C$$

$$and \quad \Psi^{b}_{t^{*}(i)}(i) \le -C$$
(9)

Such time $t^*(i)$ is called waste time according to soundness. That is to say, if the facility reaches waste time in soundness *i*, expending the waste cost and demolishing it at once are reasonable. Such waste time is different according to soundness *i*. From the definition of optimal value function,

 $\Psi_t^b(i) > \Psi_t^b(i+1)$ holds to arbitrary *t* and *i*. (i=1, ..., *I*-1). Therefore, the following monotonous relation is approved at the time of abandonment according to soundness.

$$t^{*}(1) > \dots > t^{*}(I)$$
 (10)

In addition, from the condition(8), the following is approved to an arbitrary $t < t^*(i)$.

$$V_t^b(i) = \Psi_t^b(i) \tag{11}$$

3.4. Repair policy

The repair rule $d \in D$ applied for the repair period is defined by using function that specify priority repair action in each soundness of facilities h(t)=i ($i=1,\dots,I$). Repair rules are limited, and D is set of the repair rules. The repair action ζ^d implemented under the repair rule $d \in D$ is defined as follows by using the deterioration level after the repair action:

$$\xi^{d} = \begin{pmatrix} \xi^{d}(\mathbf{I}) \\ \vdots \\ \xi^{d}(I) \end{pmatrix}$$
(12)

It is shown $c^{d}(i)$ for the repairing cost when repair action $\zeta^{d}(i)$ is adopted under the repair rule *d*. So when the soundness of facilities is improved from *i* to $\zeta^{d}(i)=l$, the repairing cost $c^{d}(i)=c_{il}$ is needed. And when the repair is not implemented, it becomes $c^{d}(i)=0$. In this regard, the repairing cost is assumed that meets the following requirement.

$$c_{ij} \leq \dots \leq c_{lj} \leq \dots \leq c_{lj}$$

($i \leq l \leq I; i = 1, \dots I; j = 1, \dots, i$) (13)

The requirement (13) means the cost for one with bad deterioration level of the repair to recover to the same deterioration level grows. Next, the transition of soundness when the repair rule $d \in D$ is implemented is defined. The content of the action based on the repair rule can be described by using the function $\zeta^d(i)$ to make the soundness $\hat{h}(t)$ after the action to the soundness h(t)=i at period tcorrespond. When repair rule d is applied at soundness i, the transitive relation of the soundness of facilities is shown by following:

$$q^{d}(i,l) = \begin{cases} 1 & \xi^{d}(i) = l \\ 0 & otherwise \end{cases}$$
(14)
(i=1,...,I; l=1,...,i)

In addition, under repair rule *d*, the repair action is implemented from h(t)=i after monitoring at the beginning of period *t*, and then, the probability that soundness will change to h(t+1)=j by the beginning of the period t+1 is shown with following:

$$\hat{\pi}_{ij}^{d} = \sum_{l=1}^{l} q^{d}(i,l)\pi_{l,j}$$
(15)

Here, it is assumed that the facility is repaired by using repair rule $d \in D$ for the beginning period *t*, and it changes soundness h(t)=j by the beginning period *t*+1. And then, discount cash flow of expected net benefit that can be acquired when the optimal management policy at each period that follow is adopted is shown $\Psi_t^c(i:d)$. The discount cash flow of expected net benefit when repair rule $d \in D$ is applied at period *t* can be shown as follows:

$$\Psi_t^c(i:d) = v(t) - c_t^d + \rho \sum_{j=1}^I \hat{\pi}_{ij}^d V_{t+1}(j:d)$$
(16)

Here, $V_{t+1}(j:d)$ shows the maximum value of discount cash flow of expected net benefit that can be acquired in the case that the optimal management policy under the repair rule $d \in D$ after the period t+1when the soundness h(t+1) = j at the beginning of period t+1(Hereafter, it is called the optimal value function).

4. APPLICATION CASES

4.1 Outline of application cases

The optimal scrapping and maintenance model proposes in this study are applied to the management problem of the bridge in Tottori Prefecture H town, and the utility of the model is verified empirically. This town is a typical intermediate and mountainous region where it is located in a Chugoku mountains, and population aging rate reaches 48%. The population of this town is about 5,000. Though the targeted bridge is comparatively short bridges of 20m in length and 8m in width, it plays the role to connect villages in this town. The geographical point of this bridge is shown in figure 1 as a pattern diagram. The bridge is constructed in the route where village A and village B are connected and the time from A to B required in the car is 20 minutes. And it faces the crisis where village A will disappear in the near future. Deterioration progresses to the bridge to some degree, and whether a necessary repair operation is implemented becomes a problem though it doesn't reach application limits at present. The circuit route shown in the dotted line can reach village B when the bridge is demolished. The time required is about 30 minutes.



figure 1 Target bridge and circuit route

4.2 Setup analysis condition

In this study, the authors give priority to the verification of the possible application of the optimal scrapping and maintenance model. When discount cash flow of expected net benefit of target bridge is calculated, a similar bridge is selected from the authors' retrospective research. Then, it refers to the calculation result of the Markov transition probability and the maintenance repair cost that authors have already estimated in this case. The target bridge is of 20m in length of the bridge and width 8m as shown in the table 1. For the waste cost to the bridge, 11,200 (1,000yen) is taken as unit price $70(1,000 \text{ yen/m}^2)$ in consideration of a concrete bridge where it did not comparatively cost the removal cost referring to past results value for the waste cost to the bridge. 450 (1,000yen/year) was summed up for the running cost for minimum maintenance cost of the patrol and the cleaning, etc. In addition, the repair is set to the repair area ratio 30%, and the repair method and the unit price concretely assumed are shown in the table. The discount cash flow of expected net benefit of target bridge is calculated based on the above-mentioned condition, and the optimal management policy is decided. And the rule repairing to each soundness

table 1 the variety of condition of the bridge

Bridge length	20m	
Width of bridge	8m	
Waste cost	11,200(1,000yen)	
Maintenance cost	450(1,000yen)	
Repair area ratio	30%	
Initial use benefit	89,192(1,000yen)	
Benefit half period	10 years	

table 2 repair method

Soundness	Repair method	Unit price	
2	Surface coating method	13(1,000yen/m ²)	
3	Clackle grouting method	50(1,000yen/m²)	
4	Steel plate bonding method	150(1,000yen/m ²)	
5	Re-decking method	350(1,000yen/m ²)	

4.3 Consideration of analysis result

sequentially was adopted as the optimal rule. The final waste time becomes 115 years under input information shown in 4.2. The final waste time means a theoretical upper limit of the life of the bridge about which thought from the aspect of the use benefit. It shows the termination point to solve scrapping and maintenance model recurrently. Actually, the phenomenon of demolishing the bridge when it is earlier than the final waste time because its deterioration progresses can happen. Under the requirement that soundness *i* was observed at beginning of period t, "waste policy", "waste reservation policy", "Repair policy" are adopted. And then, discount cash flow of expected net benefit since applicable time obtained by adopting the optimal repair rule d^* and policy, $\Psi_t^a(i)$, $\Psi_t^b(i)$, and $\Psi_t^c(i:d)$ was calculated sequentially. Discount cash flow $\Psi_t^{a}(3)$, $\Psi_t^{b}(3)$, and $\Psi_t^{c}(3:d^*)$ achieved when each policy is adopted in period t that soundness of the bridge is 3 and the soundness 3 continues period t are shown in figure 2. The policy that discount cash flow becomes the maximum is selected as an optimal policy corresponding to soundness 3 among three management policies at each period. It can be understood that the optimal policy changes from figure 2 in order of "Repair

table 3 the optimal management policy

and discount cash flow

Passed	In-service	Discount cash flow			
years	years	Soundness 2	Soundness 3	Soundness 4	Soundness 5
0	30	853,373	851,597	846,797	837,197
2	32	741,125	739,349	734,549	724,949
4	34	643,410	641,634	636,834	627,234
6	36	558,345	556,569	551,769	542,169
8	38	484,295	482,519	477,719	468,119
10	40	419,832	418,056	413,256	403,656
12	42	363,717	361,941	357,141	347,541
14	44	314,868	313,092	308,292	298,692
16	46	272,346	270,570	265,770	256,170
18	48	235,332	233,556	228,756	219,156
20	50	203,112	201,336	196,536	186,936
22	52	175,067	173,291	168,491	158,891
24	54	150,657	148,881	144,081	134,481
26	56	129,410	127,634	122,834	113,234
28	58	110,919	109,143	104,343	94,743
30	60	94,827	93,051	88,251	78,651
32	62	80,823	79,047	74,247	64,647
34	64	68,638	66,862	62,062	52,462
36	66	58,037	56,261	51,461	41,861
38	68	48,815	47,039	42,239	32,639
40	70	40,795	39,019	34,219	24,619
42	72	33,863	32,039	27,239	17,639
44	74	27,837	25,959	21,159	11,559
46	76	22,583	20,876	15,863	6,263
48	78	18,002	16,475	11,490	1,649
50	80	14,007	12,624	8,320	-2,370
52	82	10,523	9,252	5,528	-5,871
54	84	7,485	6,298	3,059	-8,920
56	86	4,837	3,707	865	-11,200
58	88	2,528	1,434	-1,096	-11,200
60	90	517	-561	-2,864	-11,200
62	92	-1,233	-2,311	-4,474	-11,200
64	94	-2,756	-3,841	-5,957	-11,200
66	96	-4,080	-5,174	-7,343	-11,200
68	98	-5,231	-6,325	-8,661	-11,200
70	100	-6,232	-7,306	-9,938	-11,200
72	102	-7,107	-8,120	-11,200	-11,200
74	104	-7,873	-8,854	-11,200	-11,200
76	106	-8,542	-9,546	-11,200	-11,200
78	108	-9,119	-10,214	-11,200	-11,200
80	110	-9,607	-10,871	-11,200	-11,200
82	112	-10,024	-11,200	-11,200	-11,200
84	114	-10,422	-11,200	-11,200	-11,200
86	116	-10,811	-11,200	-11,200	-11,200
88	118	-11,200	-11,200	-11,200	-11,200
00	1 1 2 0	-11 200	-11 200	-11 200	-11 200



figure 2 the optimal management policy (soundness 3)

policy" \rightarrow "Waste reservation policy" \rightarrow "Waste policy" along with the time passage. Because the use benefit decreases along with the time passage, the management policy of bridge changes from the repair policy into the abandonment reservation policy at the time 58 years pass from in-service even if its soundness is 3 and it changes from waste reservation policy into waste policy at the time 99

years pass from in-service.

In addition, the maximum value of discount cash flow (optimal function value) achieved when the optimal management policy would be adopted for each 114 year in the future when present is made a starting point is ordered as a stream. The result of the optimal function value is shown in the table 3 every two years. Blue in the table 3 corresponds to the repair policy. Green in the table 3 corresponds to the waste reservation policy. Red in the table 3 corresponds to the waste policy. In table 3, at the time 64 years pass, the discount cash flow is -985(1,000 yen) for soundness 5. From table 3, it can be read that years when waste policy is adopted by soundness large (Deterioration has progressed) become early, while years when waste reservation policy is adopted by soundness small (the bridge is sound) becomes long.

5. CONCLUSION

In this study, the case where infrastructures demand decreases determinately by a decrease in its demand and the progress of deterioration process in the future was assumed, and the methodology that examined the optimal waste timing and the optimal repair policy of targeted facilities is proposed. Three management policies (1) to demolish infrastructures immediately, (2) to maintain them under the minimum maintenance, and (3) to repair them constantly were taken up as management policy, and it proposed the optimal scrapping and maintenance model converted the management policy optimal in consideration of the facilities demand and the deterioration was proposed. as a result of positive analyses, it was clarified to be able to decide the optimal waste timing meeting the facilities demand and deterioration.

REFERENCES

White, D.J.: Markov Decision Process, Wiley, pp.130-146, 1993.

Eckles, J.E.: Optimal maintenance with incomplete information, Operations Research, Vol.16, pp.1058-1067, 1968.

Yoshitane, T., Kiyoyuki, K., Kazuya, A., Kiyoshi, K. :Estimating Markovian Transition Probabilities for Bridge Deterioration Forecasting, Proceedings of JSCE, No.801/I-73, pp.68-82, 2005.

Kiyoyuki, K., Keiichi, Y., Kiyoshi, K., Kei, O., : Optimal Maintenance Strategies of Bridge Components with an Average Cost Minimizing Principles, Proceedings of JSCE, No.801/I-73, pp.83-96, 2005.