OPTIMAL BRIDGE MAINTENANCE STARATEGY DTERMINED BY THE MARKOV PROCESS AND TWO STAGE OPTIMIZATION TECHNIQUE

Kazuhiro Taniwaki^{*}, Mitao Ohga^{**} and Kamal Karunananda^{***}

* Associate Professor, Department of Civil and Environmental Eng., Fukui University of Technology,
 ** Professor, Department of Civil and Environmental Eng., Ehime University
 *** Graduate Student, Department of Civil and Environmental Eng., Ehime University

ABSTRACT: In this study, the applicability of bridge deterioration forecasting method, which is developed by the Markovian transition probability model, is discussed for bridges in Fukui prefecture. Furthermore, an optimal maintenance strategy which can determine the optimal scheme of repair considering expected life-cycle cost is proposed for bridges in Fukui prefecture. The strategy is developed by using the Markovian transition probability model and two stage optimization technique. The usefulness and effectiveness of the strategy are demonstrated in the numerical example.

KEYWORDS: Markovian transition probability model, deterioration forecasting, optimum maintenance strategy, two stage optimization

1. INTRODUCTION

Recently the bridge life-span policy in Japan has been extended from fifty to one hundred years, in order to provide a more efficient use of financial resources. Local governments have therefore had to establish an optimal strategy for bridge management. Several bridge management systems have been introduced by using the Markov decision process in cooperation with optimization techniques[1]. Tsuda et al.[2] developed a bridge deterioration forecasting method using the Markovian transition probability model and the exponential hazard model[3]. The advantage of this method is that the Markovian transition probability matrix for individual bridges can be determined by considering bridge features and circumstances such as width and length of bridge, and traffic quantity. Kaito et al.[4] studied optimal maintenance strategies of bridge components based on an average cost minimizing

principle presented by Haward[5]. Recently many contributions to development of expert systems for bridge management have been made successfully by using a genetic algorithm[6-9].

In this study an optimal bridge management system is presented by using the bridge deterioration method^[2] and two optimization forecasting technique. Bridge inspection data is combined with the exponential hazard model to generate the elements of the Markovian transition probability model, which establishes future estimates for bridge deterioration. The expected life-cycle cost of a bridge is calculated by considering the repair costs corresponding to estimated repair actions. However, the maintenance strategies that prescribe when to repair, how to repair, and how many times to repair during the specified lifetime of a bridge, will be limited by yearly allocated financial resources. By considering the maintenance requirements as well as the available financial resources, life-cycle costs for

all bridges have to be minimized. To solve this complicated problem, a two stage optimization process is presented. In the first stage, the gradient projection method is used to calculate the optimal number of repair times and repair timings needed for each individual bridge, in order to provide a minimum accumulated life-cycle cost. After this stage, the time intervals between repairs for each bridge are fixed. In the second stage, the initial repair timings for all bridges are adjusted in a manner of least increase of accumulated life-cycle cost and maximum satisfaction of the constraints of yearly allocated financial resources in order to satisfy the violating constraints. Subsequent repair timings for individual bridges remain consistent with initial calculations for optimal bridge repair timings.

The proposed system is applied to the bridge deck management problems for 93 bridges in Fukui prefecture in Japan, and the usefulness and effectiveness of the strategy are illustrated in numerical examples.

2. DETERIORATION FORECASTING METHOD BY THE MARKOVIAN TRANSITION PROBABILITY MODEL

According to the Markov chain technique the future state vector $\mathbf{S}(t)$ is given by $\mathbf{S}(t) = \mathbf{S}(0)\mathbf{\Pi}^{t}$ where the past condition is irrelevant for forecasting the future condition. $\mathbf{\Pi}$ is a transition probability matrix and is expressed as

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1J} \\ 0 & \pi_{22} & \cdots & \pi_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \pi_{JJ} \end{bmatrix}$$
(1)
where $\sum_{j=1}^{J} \pi_{ij} = 1$.

 π_{ij} is the transition element which expresses the probability of bridge condition transiting from the condition rating *i* to *j*. *J* is the number of condition

Table 1 Condition rating and corresponding immediacy of action

Condition rating	Maintenance immediacy of action
1	Good condition
2	Minor structural defects without need of repair
3	Minor structural defects without urgent need of repair, but special attention
4	Showing structural defects with need of repair in order to prevent more advanced structural deterioration
5	Showing structural damage with urgent need of repair in order to guarantee safety
6	Showing serious structural damage, facility closed for repair

rating and J indicates the worst condition. In this study, the bridge condition is evaluated in the six condition ratings as shown in Table 1.

According to the bridge deterioration forecasting method using the Markovian transition probability model and the exponential hazard model[2] the element π_{ii} , where the condition rating at the initial inspection time is the degree *i* and that at the second inspection time τ years later is also the degree *i*, can be given by

$$\pi_{ii} = \exp(-\theta_i \tau) \qquad (i = 1, \dots, 5), \quad (2)$$

where θ_i is the hazard function and is also equal to $-d \log \tilde{F}_i(t_i)/dt$. In this study, θ_i is assumed as the constant $(\theta_i > 0)$ and it is recognized as the hazard ratio.

The element π_{ii+1} , where the condition rating at the initial inspection time is the degree *i* and that τ years later is the degree *i*+1, can be given by

$$\pi_{ii+1} = \frac{\theta_i}{\theta_i - \theta_{i+1}} \left\{ -\exp\left(-\theta_i\tau\right) + \exp\left(-\theta_{i+1}\tau\right) \right\}$$

$$(i = 1, \dots, 5). \tag{3}$$

The element π_{ij} $(j \ge i+2)$ is given by the following expression.

$$\pi_{ij} = -\sum_{k=i}^{5} \prod_{m=k}^{5} \frac{\theta_m}{\theta_m - \theta_j} \pi_{ik} + \prod_{m=i}^{5} \frac{\theta_m}{\theta_m - \theta_j} \exp\left(-\theta_j \tau\right)$$

$$(i = 1, \dots, 3)$$
(4)

In case of i = J, the transition element π_{iJ} can be calculated considering the necessary condition that the summation of a row in the transition probability matrix is one.

$$\pi_{iJ} = 1 - \sum_{j=1}^{5} \pi_{ij} \qquad (i = 1, \dots, 5) \qquad (5)$$

The hazard ratio θ is provided for ratings from 1 to 5 in all bridges. The hazard ratios θ_i^k $(i = 1, \dots, 5; k = 1, \dots K)$ are expressed as the following linear form using the characteristic vectors \mathbf{x}^k and those weights $\boldsymbol{\beta}$.

$$\theta_{i}^{k} = \beta_{i,1} + \beta_{i,2} x_{2}^{k} + \beta_{i,3} x_{3}^{k}$$

$$(i = 1, \dots, 5; k = 1, \dots K)$$
(6)

where *K* is the number of bridges. x_2 and x_3 are respectively the averaged deck area and traffic quantity, and those values are normalized. $\beta_{i,1}$ is a common weight set for the condition rating *i*, and $\beta_{i,2}$ and $\beta_{i,3}$ are the weights of x_2 and x_3 for the condition rating *i*, respectively. Considering the above the elements of a transition probability matrix can be obtained by calculating the optimum values of $\boldsymbol{\beta}$ instead of θ_i^k .

According to the concept of maximum likelihood method the optimum values of β can be calculated by maximizing the following likelihood function.

$$\ln[L(\boldsymbol{\beta})] = \ln\left[\prod_{i=1}^{J-1}\prod_{j=i}^{J}\prod_{k=1}^{K} \left\{\pi_{ij}\left(\boldsymbol{Z}^{k}, \boldsymbol{x}^{k}:\boldsymbol{\beta}\right)\right\}^{\overline{\delta}_{ij}^{k}}\right]$$
$$= \sum_{i=1}^{J-1}\sum_{j=i}^{J}\sum_{k=1}^{K}\overline{\delta}_{ij}^{k}\ln\left[\pi_{ij}\left(\boldsymbol{Z}^{k}, \boldsymbol{x}^{k}:\boldsymbol{\beta}\right)\right],$$
(7)

where δ_{ij}^{k} is one if the inspection result of the *k*th bridge satisfies the transition from the degree *i* to *j*, zero otherwise. In this study the likelihood function in eq.(7) is maximized with respect to β by using the gradient projection method. In the maximization process, the minimum values of β are set at 0.01 so as to satisfy the constraint of $\beta \ge 0$ and the adaptive move limit constraints, in which the largest improvements of β are limited to 20%, are imposed.

3. A NUMERICAL EXAMPLE OF DETERIORATION FORECASTING OF EXISTING BRIDGES

In this section, the results of deterioration forecasting of existing bridge decks in Fukui

Table 2 Bridge inspection data in Fukuiprefecture

Data	Age	Bridge type	Number of span (A)	Bridge length (m) (B)	Width of road (m) (C)	Averaged area of deck (C)×(B)÷ (A)	Traffic quantity per 12 hours	Condition rating	
1	6	PC·RC	7	202.0	8.6	248.2	3360	2	
2	27	PC	2	57.2	10.0	286.0	3360	2	
3	9	ST	1	33.7	11.0	370.7	269	2	
42	73	RC	1	20.0	8.0	160.0	14383	6	
91	43	RC	1	8.6	5.3	45.6	435	2	
92	43	PC	1	12.2	5.6	67.7	435	3	
93	43	RC	1	13.6	5.5	74.1	515	3	
94	31	RC	1	16.6	9.5	157.7	808	4	
95	31	PC	1	15.5	8.4	130.2	808	4	
96	31	RC	1	6.6	6.0	39.6	5458	4	
97	20	ST	1	7.1	4.5	31.7	5458	4	
98	23	RC	1	3.1	14.5	44.8	15468	4	
99	23	RC	1	4.4	8.9	39.2	3014	4	
100	43	RC	1	6.5	5.4	34.8	616	5	

prefecture shown in Table2 are investigated by adopting the method described above. The number of inspection times in the available data for 100 bridges in Table 2 is only one, therefore, the initial inspection time is set at the time of service inauguration of bridges and the initial condition ratings for all bridges are one. In this study, the interval of inspection, τ , is assumed as one year and the transition probability matrix indicates the deterioration probability distribution after one year. For the reasons that the number of available data is only 100 and the condition ratings of those bridges are mainly distributed in the range of 1-3, we may not obtain an exact transition probability matrix. Therefore, in this study, the assumed data, which indicate the transitions to the condition rating 5 in 55 years after the service inauguration and the condition rating 6 in 70 years, are added to the original data in Table 2. The bridge number 42 is the oldest one and it has been already repaired, but it is evaluated as if the repair has not be made. The lacks of covers in the decks due to poor constructions are founded for the bridge numbers 94 to 100, however, the deterioration forecasting is made including those bridges at first. The forecasting results for 93 bridges extracting those bridges are also investigated later.



Figure 1 Comparisons of condition ratings between the inspection data and the most expected forecasting results for 100 bridges

Table 3 Values of β_1, β_2 and β_3

Condition rating	β 1	β2	β3
1	4.4223	0.0653	5.8734
2	1.9990	0.01	8.4247
3	0.4358	0.01	8.3320
4	0.01	11.3580	4.6224
5	3 3 1 9 2	001	0 0213

Table 4 Transition probability matrix for bridge 42

Condition rating	1	2	3	4	5	6
1	0.9158	0.8077e-01	0.3357e-02	0.7468e-04	0.9626e-06	0.6142e-08
2	0.0	0.9207	0.7668e-01	0.2561e-02	0.4404e-04	0.3516e-06
3	0.0	0.0	0.9359	0.6250e-01	0.1613e-02	0.1718e-04
4	0.0	0.0	0.0	0.9501	0.4907e-01	0.7852e-03
5	0.0	0.0	0.0	0.0	0.9689	0.3107e-01
6	0.0	0.0	0.0	0.0	0.0	1.0

The comparison of condition ratings between the inspection data and the hazard model is shown in Figure 1. As seen from the figure, large differences in the bridges numbers 94 to 100 can be observed. This result leads that the bridges with poor constructions or poor inspections can be distinguished by adopting this forecasting proposed method. Then, the comparison of condition ratings for 93 bridges extracting the data for the bridge numbers 94 to 100 is shown in Figure 2. A similar result between the inspection data and the hazard model can be observed in total comparing to the results in figure 1. The large differences in two



Figure 2 Comparisons of condition ratings between the inspection data and the most expected forecasting results for 93 bridges



Figure 3 Deterioration transition for bridge 42 during 100 years

condition ratings can be seen for bridge numbers 14, 89 and 90. Those bridges were constructed near the coast and those may be deteriorated earlier than the expectation by the effect of salt damage. For this reason, it would need to consider the term of the effect of salt damage in eq.(6).

Table 3 shows the values of β_1, β_2 and β_3 obtained by the maximum likelihood method. The values of β_2 for condition ratings 2,3 and 5 are determined by the lower limit. This result means that the averaged deck area has nothing to do with the determinations of $\theta_2, \theta_3, \theta_5$, whereas the averaged deck area influences the determination of θ_4 greatly. The traffic quantity is the important factor for determinations of θ except for θ_5 . The elements of a transition probability matrix for bridge number 42 are shown in Tables 4 and the deterioration transmission during 100 years for the bridge number 42 is depicted in Figure 3. The largely expected probability is transited to the following worse condition ratings, i.e. the degree 2 in 12 years, the degree 3 in 22 years, the degree 4 in 36 years, the degree 5 in 51 years and the degree 6 in 67 years, respectively.

4. DETERMINATION OF OPTIMUM MAINTENANCE STRATEGIES

In this study, a determination method for optimum number of repair times and optimum repair timings, which minimize the expected life-cycle cost subject to constraints of yearly allocated financial resources, is proposed by using a two stage optimization process. In the optimization process, the transition probability matrix is transformed by using the repair transition matrix **Q**. Namely, the transition probability matrix considering the repairs, Π^r , can be expressed as $\Pi^r = \Pi \mathbf{Q}$, where

$$\mathbf{Q} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1J} \\ q_{21} & q_{22} & \cdots & q_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ q_{J1} & q_{J2} & \cdots & q_{JJ} \end{pmatrix}.$$
 (8)

If the condition rating *i* is improved up to j(j < i) by a proper repair, then the element q_{ij} takes one and others take zero. If the condition rating *i* is maintained without any repairs, then the element q_{ii} takes one and others take zero. The deck for the condition rating *J* must be replaced immediately, therefore, the elements in the *J*th row or the *J*th column always take zero.

The summation of expected life-cycle cost and indirect cost per one repair time is taken into account as the objective function. When the initial condition rating is the degree *i*, the expected life-cycle cost in one year, $u_i(1)$, is given by

$$u_i(1) = \sum_{j=i}^{J} \pi_{ij} c_j , \qquad (9)$$

where c_j is the repair cost for the condition rating j and is the *j*th element of the following repair cost vector, $\mathbf{C} = (c_1, \dots, c_j)^T$. In the case of that any repairs are not made, the values of elements except for c_j are set at zero. When the initial condition rating is the degree *i*, the expected life-cycle cost accumulated for two years, $u_i(2)$, is given by

$$u_i(2) = \sum_{j=i}^J \pi_{ij} c_j + \sum_{j=1}^J \sum_{k=j}^J \pi_{jk} \pi_{ij}^r c_k.$$
(10)

The expected life-cycle cost accumulated for three years, $u_i(3)$, is given by

$$u_{j}(3) = \sum_{j=i}^{J} \pi_{ij} c_{j} + \sum_{j=1}^{J} \pi_{ij}^{r} u_{j}(2) .$$
 (11)

In general expression, the expected life-cycle cost accumulated for t years for the initial condition rating i, $u_i(t)$, is expressed as

$$u_i(t) = \sum_{j=i}^J \pi_{ij} c_j + \sum_{j=1}^J \pi_{ij}^r u_j(t-1) = \sum_{i=1}^t \overline{u_i} , \qquad (12)$$

where \overline{u}_i is the expected cost at *i* year.

The optimization problem for determination of minimum cost maintenance strategies, which can identify the optimum number of repair times for the *k*th bridge, n_k , and the optimum repair timings for the condition rating *i* corresponding to n_k , t_{ki} , can be formulated as

find
$$n_k$$
, $t_{ki}(i=1,\dots,n_k; k=1,\dots,K)$ which
minimize $L_{CC}(\mathbf{t},\mathbf{n},\mathbf{T}) = \sum_{k=1}^{K} \left(\sum_{i=1}^{T} \overline{u}_{ki}(\mathbf{t}_k,n_k) + n_k C_r \right)$

subject to

$$g_{ki} = t_{ki+1} - t_{ki} - \bar{t} \ge 0 \quad (i = 1, \dots, n_k - 1; k = 1, \dots, K) ,$$

$$g_{ci} = \sum_{k=1}^{K} \overline{u}_{ki}(\mathbf{t}_k, n_k) - \overline{u} \le 0 \quad (i = 1, \dots, \mathbf{T})$$
(13)

where \overline{u}_{ki} is the expected cost at *i* year for the *k*th bridge and C_r is the indirect cost per one repair, respectively. \overline{t} , \overline{u} and **T** are, respectively, the allowable time intervals of adjacent repair timings,

the yearly allocated repair cost, and the maintenance term.

The numbers of repair times, $n_k (k = 1, \dots, K)$, in eq.(13) are unknown variables and the repair timings corresponding to n_k can not be determined directly. In this study, therefore, a two stage optimization process is proposed to solve the optimization problem. At the first optimization stage, aiming at the *k*th bridge n_k and t_{ki} can be determined by solving the following optimization problem.

Find n_k , t_{ki} $(i = 1, \dots, n_k)$ which minimize $L_{CC}^k(\mathbf{t}_k, n_k, \mathbf{T}) = \sum_{i=1}^{\mathbf{T}} \overline{u}_{ki}(\mathbf{t}_k, n_k) + n_k C_r$

subject to

 $g_{ki} = t_{ki+1} - t_{ki} - \bar{t} \ge 0$ $(i = 1, \dots, n_k - 1).(14)$

The above problem is solved for discretely assumed values n_k by using the gradient projection method. The optimum n_k and t_{ki} $(i = 1, \dots, n_k)$, which minimize the expected life-cycle cost accumulated in the maintenance term, can be determined by of comparing the values $L_{CC}^{k}(\mathbf{t}_{k}, n_{k}, \mathbf{T})$ for each assumed values n_{k} . The numbers of repair times and the repair timings for all bridges can be calculated in the same manner. After this stage, n_k and the time intervals between repairs for each bridge are fixed. At the second optimization stage, the initial repair timings for all bridges, t_{k1} ($k = 1, \dots, K$), are taken into account as unknown variables and the subsequent repair timings for individual bridges are improved consistently with the improvements of $t_{k1}(k=1,\dots,K)$. The optimization problem in the second optimization stage can be formulated as

find t_{k1} $(k = 1, \dots, K)$ which minimize $L_{CC}(\mathbf{t}, \mathbf{n}, \mathbf{T}) = \sum_{k=1}^{K} \left(\sum_{i=1}^{T} \overline{u}_{ki}(\mathbf{t}_{k}, n_{k}) + n_{k}C_{r} \right)$

subject to

$$g_{ci} = \sum_{k=1}^{K} \overline{u}_{ki}(\mathbf{t}_{k}, n_{k}) - \overline{u} \le 0 \quad (i = 1, \cdots, \mathbf{T}). \quad (15)$$

The optimum n_k and t_{ki} ($i = 1, \dots, n_k$), which

minimize the expected life-cycle cost accumulated in the maintenance term, is already obtained in the first optimization stage. In this stage, t_{k1} ($k = 1, \dots, K$) which satisfy the violating constraints of yearly allocated financial resources can be improved simply by sensitivity analyses in a manner of least increase of accumulated life-cycle cost and of maximum satisfaction of the constraints. The improvement element k for satisfying the most violating constraint j is determined as the following criteria.

$$r_{ck} = \min_{k} \left\{ \frac{\Delta g_{Cj}(\Delta t_{k1})}{\Delta L_{CC}(\Delta t_{k1})} \middle| \Delta g_{Cj}(\Delta t_{k1}) < 0; k = 1, \cdots, K \right\}, (16)$$

 $\Delta g_{C_i}(\Delta t_{k_1})$ and $\Delta L_{CC}(\Delta t_{k_1})$ where are, respectively, the sensitivities of g_{Ci} for the most violating constraint j and of L_{cc} with respect to $\Delta t_{k1} = \pm 1$. t_{k1} is improved as $t_{k1} + \Delta t_{k1}$. This improvement process is repeated until all constraints are satisfied. In the first optimization stage the optimum maintenance strategies for all bridges are determined by solving the expected life-cycle cost minimization problem for individual bridge. After then, the optimum maintenance strategies are improved by simple sensitivity analyses so as to satisfy the yearly allocated available financial resources. Therefore, the proposed method can determine the optimum maintenance strategies for all bridges quite efficiently without relation to the number of bridges to be dealt with. This is a great advantage of the proposed method.

5. NUMERICAL EXAMPLES

In this section, the determination method of optimum maintenance strategies is applied to the inspection data for 93 bridges shown in Table 2. Repair action and repair cost for each condition rating are shown in Table 5. Figure 4 shows the relationship between the expected life-cycle cost and the number of repair times for bridge 42 in Table 2. The optimum repair timings and repair cost for each number of repair times for bridge 42 are summarized

Ta	ble 5	Repair	works	and	repair	costs
						Improve

Actions	Present rating	Repair works	Repair cost	Improved condition rating
Action1	2	Surface coating method	10,000 yen/ m ⁶	1
Action2	3	Crack injection method	30,000 yen/ m	2
Action3	4	Fiber seat method	50,000 ven/m	2
Action4	5	Epoxy bonded steel plate	120,000yen/ m	2
Action5	6	Slab Replacement	380,000yen/ m	1



Figure 4 Relationship between the expected life-cycle cost and the number of repair times

No of		Expected life-	Indirect	Total
ropair	Ontimum renair timings	cycle cost	expenses	expected life-
timos	optimum ropair timings	due to repairs	due to	cycle cost
times		(x1000yen)	repairs	(x1000yen)
0		104345.6	0	104345.6
1	51	94993.6	1500	96493.6
2	30, 55	89816.0	3000	92816.0
3	23, 41, 62	86496.0	4500	90996.0
4	19, 34, 46, 64	84387.2	6000	90387.2
5	16, 27, 38, 49, 64	82809.6	7500	90309.6
6	13, 22, 32, 43, 52, 65	81574.4	9000	90574.4
7	11, 19, 28, 36, 45, 54, 65	80582.4	10500	91082.4
8	9, 16, 23, 32, 40, 48, 56, 65	79768.0	12000	91768.0
9	7, 14, 21, 28, 35, 42, 50, 57, 66	79076.8	13500	92576.8
10	4, 11, 18, 25, 32, 39, 46, 53, 60, 67	78569.6	15000	93569.6

Table 6 Optimum repair timings and repair cost for each number of repair times

in Table 6. In the numerical example, the initial condition rating and the management term **T** are respectively set at the degree 1 and 100 years. C_r and \bar{t} are respectively assumed as 1.5×10^6 yen and 7 years.

As seen from Figure 4 the expected life-cycle cost decreases as the number of repair times increases, however, the summation of expected life-cycle cost and indirect cost is minimized at number 5 in this problem. In a investigation of the optimum repair timings, the initial repair is required 16 years later after the service inauguration and the condition rating is improved from the degree 2 to 1. After this the deck of bridge is repaired in cycles of 11-15 years. During last 36 years in the management term the bridge is remained without any repairs and

the final expected condition rating is the degree 4.

The distribution of total expected life-cycle cost for each year, in which an individual expected life-cycle cost for 93 bridges is taken from the results of optimum strategies obtained in the first optimization stage considering the initial condition ratings in Table 2, is depicted in Figure 5. Figure 7 shows the distribution of total expected life-cycle cost for each year after improvement in the second optimization stage. The yearly allocated financial resources are limited to 5.0×10^8 yen. The largest repair cost is required at the first year as seen from Fig.7, however, after 173 iterations in the second optimization stage all constraints are satisfied and the repair costs are distributed averagely. The increment of total expected life-cycle cost in the



Figure 5 Distribution of total expected life-cycle cost for each year obtained in the first optimization stage

management term **T** is only 0.88% compared to that in the first optimization stage.

6. CONCLUSIONS

The following conclusions can be drawn from this study.

- The Markovian transition probability model and the exponential hazard model can estimate the bridge deterioration precisely by deleting the data of the bridges with poor constructions or poor inspections.
- The term of the effect of salt damage need to be considered in the characteristic vector in addition to the averaged deck area and traffic quantity.
- 3) The proposed two stage optimization process can determine optimum maintenance strategies which minimize the expected life-cycle costs for all bridges subject to the maintenance requirements as well as the available financial resources.
- 4) In the first optimization stage the optimum maintenance strategies for all bridges are determined by solving the expected life-cycle cost minimization problem for individual bridge. After then, the optimum maintenance strategies are improved by simple sensitivity analyses so as to



Figure 6 Distribution of total expected life-cycle cost for each year obtained in the second optimization stage

satisfy the yearly allocated available financial resources. Therefore, the proposed method can determine the optimum maintenance strategies for all bridges quite efficiently without relation to the number of bridges to be dealt with.

5) In a numerical example in which the management term is assumed at 100 years, the initial repair is required at 16 years later after the service inauguration and the condition rating is improved from the degree 2 to 1. After this the deck of bridge is repaired in cycles of 11-15 years. During last 36 years in the management term the bridge is remained without any repairs.

REFERENCES

- YANEV,B., 2007. Bridge Management, John Wiley & Sons
- Tsuda, Y., Kaito, K., Aoki, K. and Kobayashi, K., 2005. Estimating Markovian Transition Probabilities for Bridge Deterioration Forecasting, *J. of Structural Mechanics and Earthquake Engineering*, No.801/I-73, pp.69-82 (in Japanese)
- 3. Lancater, T., 1990. The Econometric Analysis of

Transition data, Econometric Society Monographs, Cambridge University Press

- Kaito,K., Yasuda,K., Kobayashi,K. and Owada,K., 2005. Optimal Maintenance Strategies of Bridge Components with an Average Cost Minimizing Principles", *J. of Structural Mechanics and Earthquake Engineering*, No.801/I-73, pp.83-96 (in Japanese)
- 5. Howard,R.A., 1964. *Dynamic programming and Markov processes*, M.I.T. Press
- Nakamura,H., Kawamura,K., Onimaru,H. and Miyamoto,A., 2001. Optimization of Bridge Maintenance Strategies by Using GA and IA Techniques, *J. of Structural Engineering*, Vol.47A, pp.201-210 (in Japanese)
- Chikata,Y., Shimizu,H. and Hirose,A., 2001. A Study on GA Applied Bridge Repair Planning with the Virus Theory of Evolution, *J. of Structural Engineering*, Vol.47A, pp.211-219 (in Japanese)
- Furuta,H., Yokota,T. and Dogaki,M., 2003. Application of Fuzzy Learning and Classifier to Bridge Maintenance Management System, *J. of Structural Engineering*, Vol.49A, pp.233-243 (in Japanese)
- Jha,M.K. and Abdullah,J., 2006. A Markovian approach for optimizing highway life-cycle with genetic algorithms by considering maintenance of roadside appurtenances, *J. of the Franklin Institute*, Vol.343, pp.404-419