

# REGISTRATION FOR DIGITAL CAMERA IMAGE OF PAPER MAP DISTORTED BY TRANSPARENT FILM

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**ABSTRACT:** In the past, a large accurate image scanner was used for map digitizing. The image scanner is so expensive that it is difficult to use in general. Nowadays, a digital camera with 12 mega pixels can be used in many fields. In the future, such digital camera will be able to replace the image scanner. The digital camera will become suitable equipments for collecting many GIS data by map digitizing. But registration accuracy of digital map by the digital camera will be still lower than by the image scanner. For example, a paper map is needed to press by a transparent film for correction of wrinkle. Then the digital camera image distorted by transparent film should transformed for registration. In this study, geometric transformation was established by extended collinearity equation with snell's law for correction of refractive index. This geometric transformation was evaluated by using simulated data. This method can be applied in the case of distortion by accurate optical glass.

## 1 Background

In the past, a large accurate image scanner was used for map digitizing. The image scanner is so expensive that it is difficult to use in general. Nowadays, a digital camera with 12 mega pixels can be used in many fields. In the future, such digital camera will be able to replace the image scanner. The digital camera will become suitable equipments for collecting many GIS data by map digitizing. But registration accuracy of digital map by the digital camera will be still lower than by the image scanner. For example, a paper map is needed to press by a transparent film for correction of wrinkle. Then the digital camera image of the paper map which covered by transparent film should transformed for registration. By using two-dimensional perspective projection, the registration accuracy will become lower due to refractive index of transparent film. When error due to refractive index can be eliminated, registration accuracy will be improved.

## 2 Objectives

Objective of this study is development of registration method for digital camera image of paper map distorted by transparent film. Geometric transformation was established by extended collinearity equation with snell's law for correction of refractive index. Figure1 shows relation between incidence angle and distorted value using snell's law when refractive index is 2 and thickness of transparent film is 1mm. Figure 2 shows relation between pixel number of digital camera image and distorted value in same condition with previous figure. The pixel number is corresponded with incidence angle. In this case, angle of one pixel in calibrated digital camera image is 0.00026551 deg. Therefore, when accurate registration such as less than 0.1mm is

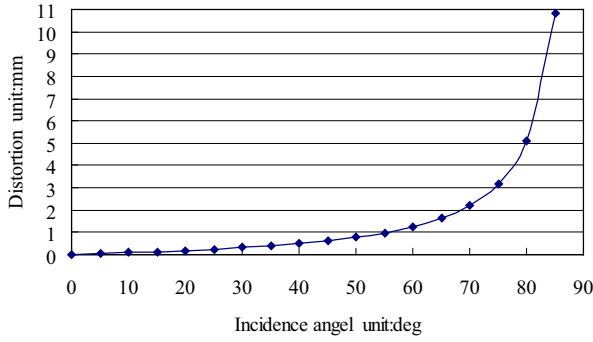


Figure 1: Relation between incidence angle and distorted value

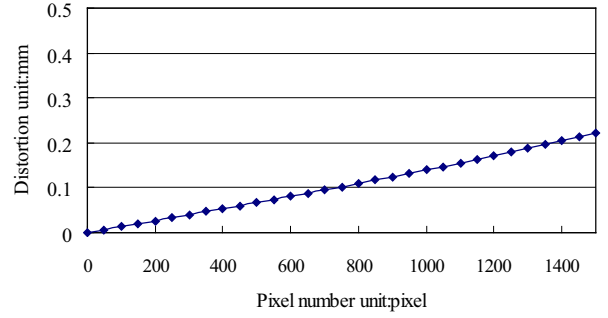


Figure 2: Relation between pixel number and distorted value

required, distortion of refractive index must be corrected.

## 3 Geometric Transformation

A conceptual procedure for registration of digital camera image is illustrated in Figure 3 and Figure 4. In this study, the paper map is assumed XY plane of object space. First, distortion value of control points by refractive index is calculated by using Snell's law with equation (1).

$$\begin{cases} \Delta X_p = \Delta D_p \cos \kappa_p \\ \Delta Y_p = \Delta D_p \sin \kappa_p \end{cases} \quad (1)$$

$\Delta D_p$  means distorted value.  $\Delta X_p$  and  $\Delta Y_p$  are distortion component of control points in map coordinates.  $\kappa_p$  is rotation angle of control points around  $Z$  axis of object space.  $\kappa_p$  can be calculated following equation.

$$\kappa_p = \tan^{-1} \frac{Y_i - Y_0}{X_i - X_0}$$

$X_i$ ,  $Y_i$  and  $Z_i$  are coordinates of imaging control point in object space.  $X_0$ ,  $Y_0$  and  $Z_0$  are coordinates of projection center in object space. Distortion value can be calculated by following equation using snell's law.

$$\Delta D_p = thickness(\tan \theta_i - \tan \theta_r)$$

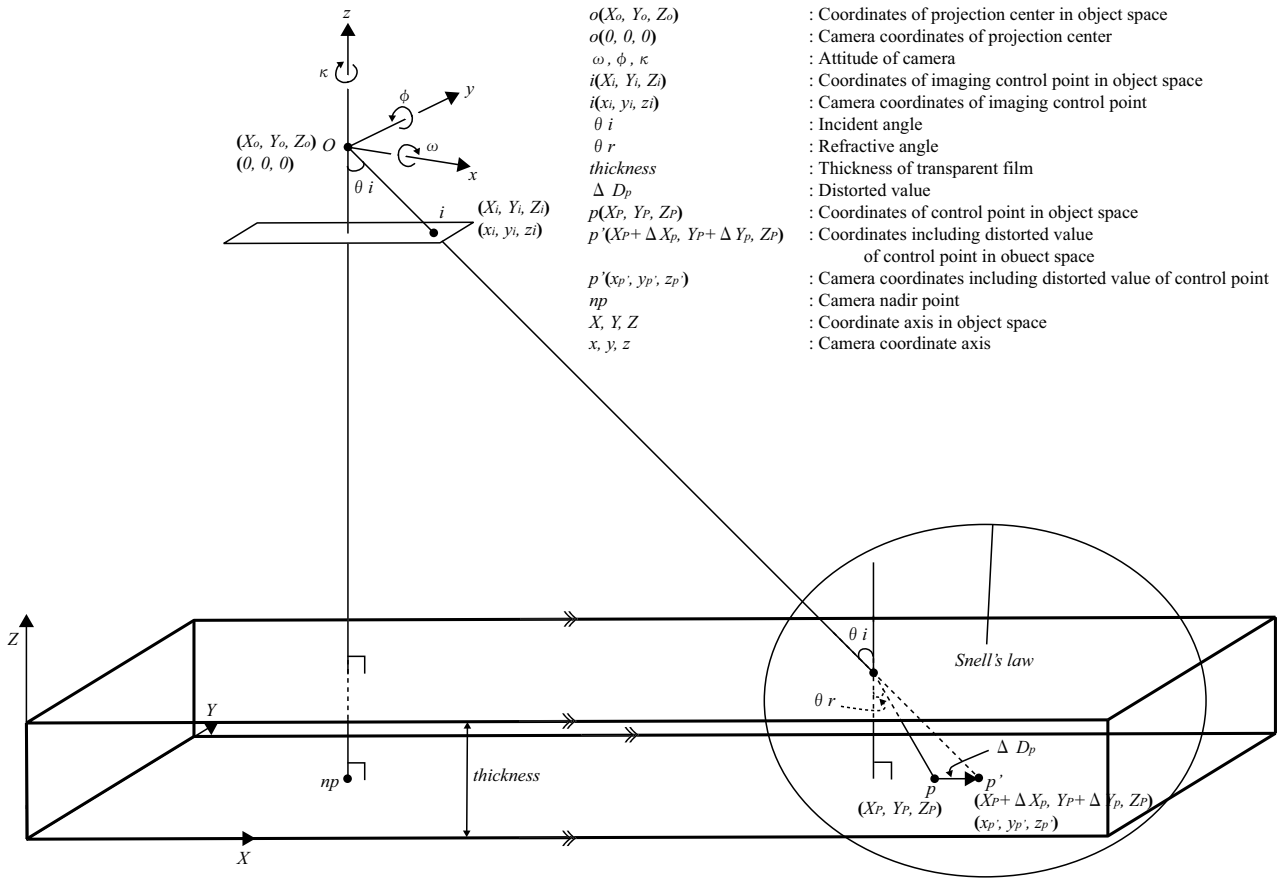


Figure 3: Principle of registration for digital camera image with transparent film

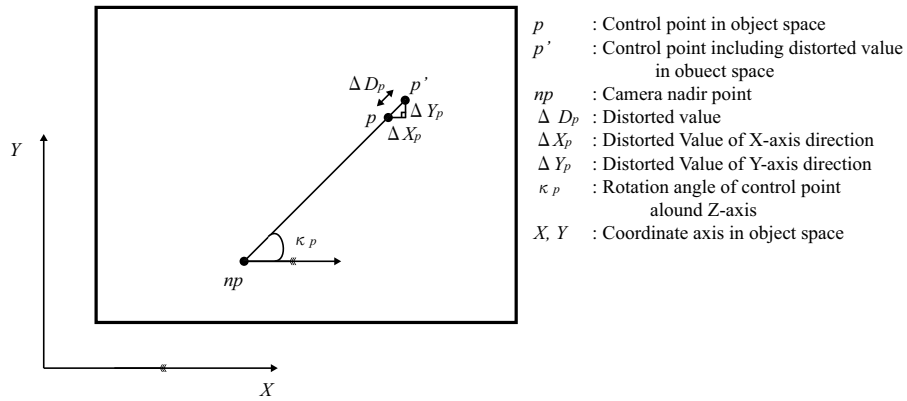


Figure 4: Projected Figure in X-Y plane

$$\theta_i = \cos^{-1} \frac{Z_i - Z_0}{c}$$

$$\theta_r = \sin^{-1} \frac{\sin \theta_i}{n_{ir}}$$

*thickness* is thickness of transparent film.  $\theta_i$  is incident angle.  $\theta_r$  is refractive angle.  $n_{ir}$  is refractive index of transparent film.

$$n_{ir} = \frac{\sin \theta_i}{\sin \theta_r} \quad (\text{Snell's law})$$

$X_i - X_0$ ,  $Y_i - Y_0$  and  $Z_i - Z_0$  can be calculated by following equation.  $x_i$ ,  $y_i$  and  $c$  are camera coordinates of control points.  $\omega$ ,  $\phi$  and  $\kappa$  are the attitude of the digital camera.

$$\begin{pmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ -c \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & -\sin \omega \\ 0 & \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \kappa & -\sin \kappa & 0 \\ \sin \kappa & \cos \kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Usually, collinearity equation can be derived from this equation. In this study, this equation was extended to solve distortion of refractive index as follows.

$$\begin{pmatrix} x_{p'} \\ y_{p'} \\ z_{p'} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X_p + \Delta X_p - X_0 \\ Y_p + \Delta Y_p - Y_0 \\ Z_p - Z_0 \end{pmatrix}$$

$$\begin{cases} x_i = -c \frac{x_{p'}}{z_{p'}} \\ y_i = -c \frac{y_{p'}}{z_{p'}} \end{cases} \quad (2)$$

where  $x_i$  and  $y_i$  are camera coordinates of control points;  $c$  is focus length;  $X_p$ ,  $Y_p$  and  $Z_p$  are coordinates of control points in object space. In this extended collinearity equation, unknown parameters are  $X_0$ ,  $Y_0$ ,  $Z_0$ ,  $\omega$ ,  $\phi$ ,  $\kappa$  and  $n_{ir}$ . To derive these unknown parameters, control point data should be used. At least 4 control point data are needed. When control point data  $(X_p, Y_p, Z_p)$   $(x_i, y_i)$  input to equation(2), least square method is applied to derive unknown parameters. This extended collinearity equation is also non linear function(Equation(3)), that must be expanded in Taylor series to make linear function.

$$\begin{cases} F(X_0, Y_0, Z_0, \omega, \phi, \kappa, n_{ir}) = -c \frac{x_{p'}}{z_{p'}} - x_i \\ G(X_0, Y_0, Z_0, \omega, \phi, \kappa, n_{ir}) = -c \frac{y_{p'}}{z_{p'}} - y_i \end{cases} \quad (3)$$

Partial differential coefficients in the Taylor series shows following equations.

$$\frac{\partial F}{\partial X_0} = \frac{ca_{11} + x_i a_{31}}{z_{p'}}$$

$$\frac{\partial F}{\partial Y_0} = \frac{ca_{12} + x_i a_{32}}{z_{p'}}$$

$$\frac{\partial F}{\partial Z_0} = \frac{ca_{13} + x_i a_{33}}{z_{p'}}$$

$$\frac{\partial F}{\partial \omega} = \frac{x_i y_i}{c}$$

$$\frac{\partial F}{\partial \phi} = -\frac{x_i^2}{c} \cos \omega - y_i \sin \omega - c \cdot \cos \omega$$

$$\frac{\partial F}{\partial \kappa} = \frac{\partial F}{\partial X_0} (Y_p + \Delta Y_p - Y_0) - \frac{\partial F}{\partial Y_0} (X_p + \Delta X_p - X_0)$$

$$\frac{\partial F}{\partial n_{ir}} = \frac{(-ca_{11} - x_i a_{31}) \frac{\partial \Delta X_p}{\partial n_{ir}} + (-ca_{12} - x_i a_{32}) \frac{\partial \Delta Y_p}{\partial n_{ir}}}{z_{p'}}$$

$$\frac{\partial G}{\partial X_0} = \frac{ca_{21} + y_i a_{31}}{z_{p'}}$$

$$\frac{\partial G}{\partial Y_0} = \frac{ca_{22} + y_i a_{32}}{z_{p'}}$$

$$\frac{\partial G}{\partial Z_0} = \frac{ca_{23} + y_i a_{33}}{z_{p'}}$$

$$\frac{\partial G}{\partial \omega} = c + \frac{y_i^2}{c}$$

$$\frac{\partial G}{\partial \phi} = x_i \sin \omega - \frac{x_i y_i}{c} \cos \omega$$

$$\frac{\partial G}{\partial \kappa} = \frac{\partial G}{\partial X_0} (Y_p + \Delta Y_p - Y_0) - \frac{\partial G}{\partial Y_0} (X_p + \Delta X_p - X_0)$$

$$\frac{\partial G}{\partial n_{ir}} = \frac{(-ca_{21} - y_i a_{31}) \frac{\partial \Delta X_p}{\partial n_{ir}} + (-ca_{22} - y_i a_{32}) \frac{\partial \Delta Y_p}{\partial n_{ir}}}{z_{p'}}$$

$$\frac{\partial \Delta X_p}{\partial n_{ir}} = -(\cos \kappa_p) \text{thickness} \frac{\partial \tan \theta_r}{\partial n_{ir}}$$

$$\frac{\partial \Delta Y_p}{\partial n_{ir}} = -(\sin \kappa_p) \text{thickness} \frac{\partial \tan \theta_r}{\partial n_{ir}}$$

$$\frac{\partial \tan \theta_r}{\partial n_{ir}} = \frac{-\sin \theta_i}{\cos^2 \left( \sin^{-1} \frac{\sin \theta_i}{n_{ir}} \right) \sqrt{1 - \left( \frac{\sin \theta_i}{n_{ir}} \right)^2} n_{ir}^2}$$

When initial condition of unknown parameters must be decided, correction value for initial condition can be calculated by least square method with expanded equation(3). By using corrected parameters, correction value can be calculated again. This calculation should be iterated to converge enough accuracy. The program in C language was coded to calculate unknown parameters based on previous procedure.

## 4 Evaluation by a simulation

Simulated data those were  $(X_p, Y_p, Z_p)$  and  $(x_i, y_i, z_i)$  were generated for evaluation. The simulated data can be used as control point. When parameters those were  $X_0, Y_0, Z_0, \omega, \phi, \kappa$  and  $n_{ir}$  were decided,  $(X_p, Y_p, Z_p)$  and  $(x_i, y_i, z_i)$  can be calculated by equation(1) and (2). In this evaluation, 9 control points were generated. Table 1 shows results of iterative solution using 9 control points of simulated data. Simulated data were  $(X_p, Y_p, Z_p)$  and  $(x_i, y_i, z_i)$ . In simulated data,  $\omega, \phi$  and  $\kappa$  were  $0^\circ$ ,  $X_0$  and  $Y_0$  were  $0mm$ ,  $Z_0$  was  $700mm$  and refractive index was 2. The result shows that unknown parameter of refractive index was solved by around 10 times iterative solution when initial condition of refractive index was input over 1.2.

Table 1: Results of iteration solution

Initial condition of refractive index	Iteration number	Calculated results
1.0	-	divergence
1.2	13	1.999976
1.4	12	1.999976
1.6	11	1.999976
1.8	11	1.999976

## 5 Conclusion

Registration method for digital camera image of paper map distorted by transparent film was successfully developed. This method can be applied in the case of the distortion by accurate optical glass. When a soft plastic film is covered on the paper map, the distortion will not be eliminated because of undulation on the surface. This registration method must be demonstrated in real paper map covered by optical glass and plastic film in near future.

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